Sample Mathematics 106 Questions

(1) Calculate $\lim_{x\to 5} \dfrac{x^2+8x-65}{x-5}$.

(2) Consider an object moving in a straight line for which the distance s (measured in feet) it's travelled from its starting point is given by the formula $s = 0.1t^2 + 10t$, where t represents time (measured in seconds).
   (a) Find the average speed of the object during the time period $2 \le t \le 5$.
   (b) Find the instantaneous speed of the object when $t = 3$.

(3) Find $\dfrac{d}{dx}\left(x^3 - 5x^2 + 8e^x + \ln x\right)$.

(4) Let $f(x) = x^2e^x$. Find $f'(x)$.

(5) Let $g(x) = \dfrac{\ln x}{1+x^2}$. Find $g'(x)$.

(6) Let $h(x) = (x^4+5)^{10}$. Find $h'(x)$.

(7) Consider a function $f(x)$ for which $f'(x) = \dfrac{(x-8)(x+2)}{x}$. Analyze its monotonicity. *That is, find the intervals on which f is increasing and those on which f is decreasing.*

(8) Find all local extrema for the function $f(x) = x^3 + 3x^2 - 9x + 5$.

(9) Use an analysis of the monotonicity and concavity of $f(x) = x^2 - 10x + 20$ to sketch its graph.

(10) Consider a function f which is continuous and differentiable everywhere, increasing on $(-\infty, 0) \cup (10, \infty)$, decreasing on $(0, 10)$, concave up on $(5, \infty)$, concave down on $(-\infty, 5)$, and for which $f(0) = 10$, $f(5) = 3$ and $f(10) = -4$. Find and identify all local extrema and sketch its graph.

(11) Find two numbers whose sum is 24 such that the product of the first with the square of the second is maximal.

(12) Suppose that tests on a fossil show that 80% of its carbon-14 has decayed. Estimate the age of the fossil, assuming that carbon-14 has a half-life of 5750 years.

(13) Suppose $750 is placed in a bank account paying interest at an annual rate of 4.5% and left undisturbed to accumulate. What will the balance be after ten years (a) if interest is compounded quarterly and (b) if interest is compounded continuously?

(14) Calculate $\int x^2 - 5x + 8\, dx$.

(15) Calculate $\int_1^3 x^3\, dx$.

(16) Calculate $\lim_{x\to 5} \dfrac{x^2+8x-65}{x-5}$.

(17) Consider an object moving in a straight line for which the distance s (measured in feet) it's travelled from its starting point is given by the formula $s = 0.1t^2 + 10t$, where t represents time (measured in seconds).
   (a) Find the average speed of the object during the time period $2 \le t \le 5$.
   (b) Find the instantaneous speed of the object when $t = 3$.

(18) Let $f(x) = x^2e^x$. Find $f'(x)$.

(19) Let $h(x) = (x^4+5)^{10}$. Find $h'(x)$.

(20) Consider a function $f(x)$ for which $f'(x) = (x-8)(x+2)$.

(a) Analyze its monotonicity. *That is, find the intervals on which f is increasing and those on which f is decreasing.*

(b) Find where its local extrema occur and identify each as either a local minimum or a local maximum.

(21) Consider a function f which is continuous and differentiable everywhere, increasing on $(-\infty, 0) \cup (10, \infty)$, decreasing on $(0, 10)$, concave up on $(5, \infty)$, concave down on $(-\infty, 5)$, and for which $f(0) = 10$, $f(5) = 3$ and $f(10) = -4$. Find and identify all local extrema and sketch its graph.

(22) Find two numbers whose sum is 60 such that the product of the first with the square of the second is maximal.

(23) Suppose a product has demand function $p = 30 - q$, where p is the price and q its demand.

(a) Graph the demand function.

(b) Find a formula for the revenue function $R(q)$.

(c) Find a formula for the marginal revenue.

(d) At what price is the marginal revenue equal to 0?

(24) Find the area of the region above the $x-$axis, between the vertical lines $x = 2$ and $x = 4$ and below the curve $y = x^3$.

(25) A car is going in a straight line at a speed of 80 feet per second when the brakes are applied, slowing the car down at a rate of 10 feet per second each second.

(a) Find a formula for its speed v, in feet per second, in terms of the time t in seconds elapsed since the brakes were applied.

(b) Find a formula for the distance s, in feet, travelled since the brakes were applied. This should also be in terms of t.

(c) How long does it take for the car to stop?

(d) How far does the car go before it stops?

(26) Consider the function $f(x) = \begin{cases} 10 & \text{when } x < 4 \\ 3x + 2 & \text{when } x \geq 4. \end{cases}$

(a) Sketch its graph.

(b) Find $f(-4)$.

(c) Find $f(0)$.

(d) Find $f(2)$.

(e) Find $f(4)$.

(f) Find $f(6)$.

(g) Find $\lim_{x \to 2} f(x)$.

(h) Find $\lim_{x \to 6} f(x)$.

(i) Suppose $g(x) = f(x - 4)$. Sketch the graph of g.

(27) Consider the function $f(x) = x^2$.

(a) Sketch its graph.

(b) Let P be the point on the graph of f whose first coordinate is 3. Find the second coordinate of P.

(c) Let Q be the point on the graph of f whose first coordinate is 4. Find the second coordinate of Q.

(d) Find the slope of the line segment $\overline{PQ}$ connecting P and Q.
(e) Find a general formula for the slope of the line segment joining the point $(5, 25)$ on the graph of f with an arbitrary point (x, x^2) on the graph of f.
(f) Simplify the above formula as much as possible.
(g) Suppose $g(x)$ represents the slope of the line segment joining the point $(5, 25)$ on the graph of f with an arbitrary point (x, x^2) on the graph of f. *Note that in the previous two questions you actually developed a formula for $g(x)$.* Calculate $\lim_{x\to 5} g(x)$.

(28) Suppose the height of an object dropped from an unknown height is given by the formula $h = 64 - 16t^2$, where t represents the time, measured in seconds, elapsed since the object was dropped and h represents the height of the object measured in feet. Also, let $v_{\text{av}}[a, b]$ represent the average speed of the object during the time period $a \leq t \leq b$.
(a) From what height was the object dropped? *Hint: Think of when the object was dropped.*
(b) How many seconds did it take for the object to reach the ground? *Hint: Think of how high the object will be when it hits the ground.*
(c) Find $v_{\text{av}}[1, 2]$.
(d) Find a general formula for $v_{\text{av}}[1, t]$.
(e) Simplify the above formula as much as possible.
(f) Calculate $\lim_{t\to 1} v_{\text{av}}[1, t]$.

(29) Find the following limits.
(a) $\lim_{x\to 5} 4x + 3$
(b) $\lim_{x\to 2} \dfrac{x^2 - 4}{x - 2}$
(c) $\lim_{x\to 1} \dfrac{f(x) - f(1)}{x - 1}$ if $f(x) = 3x + 2$.

(30) Consider the function $f(x) = x^2$ and use the definition $f'(c) = \lim_{x\to c} \dfrac{f(x) - f(c)}{x - c}$ to show that $f'(8) = 16$. *Note: You may* **not** *use the power rule to show that $f'(x) = 2x$ and thus $f'(8) = 2 \cdot 8 = 16$.*

(31) Consider the graph of the equation $y = x^2 + 3x - 2$. Find the slope of the line which is tangent to the graph at the point $(3, 16)$.

(32) Suppose the formula $s = 2t^4 + 5t^2$ gives the distance s, measured in feet, that an object has moved in a straight line from a given point in time t, measured in seconds.
(a) Find a formula for the velocity v.
(b) Find the velocity when $t = 2$.

(33) Consider the function $f(x) = x^2 + 3\ln(x) - 7e^x$.
(a) Use your calculator to estimate $f(2)$. In other words, obtain a calculator estimate for $2^2 + 3\ln(2) - 7e^2$.
(b) Obtain a formula for $f'(x)$.

(34) Find $\dfrac{d}{dx}\left((x^2 + 1)e^x\right)$.

(35) Let $f(t) = \dfrac{\ln(t)}{e^t}$. Obtain a formula for $f'(t)$.
(36) Let $g(x) = (x^4 + 3x)^5$. Obtain a formula for $g'(x)$.
(37) Let $h(x) = e^{3x+\ln(x)}$. Obtain a formula for $h'(x)$.
(38) Suppose \$250 is placed in an account paying interest at an annual rate of 3%.
  (a) Suppose interest is compounded quarterly (that is, four times per year).
    (i) What is the balance after three years?
    (ii) What is the effective annual yield?
  (b) Suppose interest is compounded continuously.
    (i) What is the balance after three years?
    (ii) What is the effective annual yield?
(39) Consider a function $y = f(x)$ for which the following information is known.
  - Its graph is continuous and smooth everywhere.
  - f is increasing on $(-\infty, -5) \cup (2, \infty)$.
  - f is decreasing on $(-5, 2)$.
  - Its graph is concave up for $x > 0$.
  - Its graph is concave down for $x < 0$.
  - The following values of the function are known: $f(-5) = 8$, $f(0) = 0$, and $f(2) = -3$.

  (a) Sketch a graph of the function f.
  (b) Does f have any local minima? If so, what are those values and for what values of the independent variable are they taken on?
  (c) Does f have any local maxima? If so, what are those values and for what values of the independent variable are they taken on?
  (d) At what point(s) does the concavity of the graph change? *Note that these points are known as* **points of inflection.**

(40) Consider the function f defined as follows: $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ 8 & \text{when } 0 \leq x \leq 5 \\ 2x + 1 & \text{when } x > 5. \end{cases}$
  (a) Sketch the graph of f.
  (b) Find $f(3)$.
  (c) Find $f(0)$.
  (d) Find $f(-10)$.
  (e) Find $f(10)$.
  (f) Let $g(x) = f(x - 3)$. Sketch the graph of g.
(41) Suppose \$500 is placed in a bank account paying interest at an annual rate of 3.5%. What is the balance after six (6) years if (a) interest is compounded semi-annually and (b) if the interest is compounded continuously?
(42) Calculate each of the following limits.
  (a) $\lim_{x \to 5} 4x + 3$
  (b) $\lim_{h \to 0} \dfrac{h^2 + 3h}{h}$

(c) $\lim_{x\to 3} \dfrac{x^2-9}{x-3}$

(43) Consider the function f defined by the formula $f(x) = 3x - 2$. Evaluate and simplify the following.
(a) $f(4)$
(b) $f(w)$
(c) $f(w+2)$
(d) $f(x+2)$
(e) $\dfrac{f(x)-f(4)}{x-4}$

(44) Sketch graphs of each of the following functions.
(a) $y = x^2$
(b) $y = (x-3)^2$
(c) $y = (x-3)^2 + 5$
(d) $y + 5 = (x-3)^2$

(45) Suppose $500 is placed in a financial instrument paying interest at an annual rate of 4%, compounded continuously.
(a) What will the balance be after 5 years?
(b) How long will it take before the total amount of interest earned reaches $200?

(46) Consider an object which is moving in a straight line and for which the distance it has moved is given by the formula $s = t^2 + 2t$, where t represents time, measured in seconds, and s represents the distance it has moved.
(a) How far does it move in the time period $2 \le t \le 4$?
(b) What is its average speed during that time period?
(c) Find a formula for its average speed $v_{\text{ave}}[2, 2+h]$ over an arbitrary time period $2 \le t \le 2+h$.
(d) Find $\lim_{h\to 0} v_{\text{ave}}[2, 2+h]$.

(47) Use a calculator to evaluate $\dfrac{4 \cdot 3^{\ln 5} + e^2}{11}$.

(48) Use the definition $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to show that $f'(4) = 5$ if $f(x) = x^2 - 3x + 4$.

(49) Calculate $\dfrac{d}{dx}(5x^2 + 2\ln x - 7e^x)$.

(50) Let $g(x) = (\ln x)^2$. Find $g'(x)$.

(51) Let $g(t) = \dfrac{t+1}{t-1}$. Find $g'(2)$.

(52) Let $f(x) = \dfrac{x^2}{\ln x}$. Find $f'(x)$.

(53) Let $y = \sqrt{3x^2 + x + 7}$. Find y'.

(54) Let $f(t) = t^2 \ln t$. Find $f'(t)$.

(55) Let $g(x) = (x^2 + x + 1) \cdot (x^2 - x + 1)$. Find $g'(x)$.

(56) Let $f(x) = x^2 - 8x + 7$. Determine all intervals on which $f'(x)$ is positive and all intervals on which $f'(x)$ is negative.

(57) Use your calculator to evaluate the following expression *without writing down any intermediate calculations.* Include at least 8 significant digits in your result, which should be written down in ordinary decimal notation. *Do not use scientific notation.* Also, write down the *exact* sequence of keys that you pressed to obtain your result.

$$\frac{2^{3.7} + 5\ln(2.1)}{3e^2 + 1}$$

(58) Calculate $\lim_{x\to 5} \dfrac{x^2 + 8x - 65}{x - 5}$.
(59) Let $f(x) = x^3 - 5x^2 + 2x + 17 - 2e^x - 5ln(x)$. Find $f'(x)$.
(60) Let $g(x) = \dfrac{x^2 + 3x + 1}{2x - 5}$. Find $g'(x)$.
(61) Let $f(t) = t^2\sqrt{4t + 9}$. Find $f'(t)$.
(62) Suppose the total cost of producing x units of a particular commodity is $C(x) = \dfrac{2}{5}x^2 + 3x + 10$. What is the marginal cost?
(63) Analyze the monotonicity of the function $f(x) = x^2 + 10 - 5$.
(64) Analyze the concavity of the graph of the function $y = x^3 - 21x^2 + 8x - 5$.
(65) Consider a function $f(x)$ with the following properties.
  (a) f is increasing for $x < -3$ and for $x > 5$.
  (b) f is decreasing for $-3 < x < 5$.
  (c) The graph is concave up for $x > 2$.
  (d) The graph is concave down for $x < 2$.
  (e) $f(-3) = 10$, $f(2) = 7$, $f(5) = 2$.
  Sketch the graph of f and identify all local extrema.
(66) A farmer wishes to enclose a rectangular field of area 1800 square feet using an existing wall as one of its sides. The cost of the fence for the other three sides is \$5 per linear foot. Find the dimensions of the rectangular field that minimizes the cost of the fence.
(67) Find $\int x^2 + \dfrac{3}{x}\,dx$.
(68) Find $\int x(x^2 + 5)^3\,dx$.
(69) Sketch the plane region bounded by the curve $y = x^3$, the x-axis and the line $x = 4$ and find its area.
(70) Suppose the speed of a car making a jackrabbit start from a red light is given by $v = 2t + 20$ for $0 \le t \le 4$, where v represents the car's speed in feet per second and t represents the time, in seconds, measured from the moment the light turns green. How far does the car go in those 4 seconds?
(71) Calculate $\int_1^5 x^3 + 3\,dx$.
(72) Calculate the following expression. Perform the calculation without writing down any intermediate calculations or using the memory locations of your calculator. Write down the brand and model of your calculator along with the exact sequence of keys your pressed to obtain your result. Write down the result of your calculation in standard decimal notation; *do not use scientific notation.*

$$\frac{3e^{4.1} + 3\sqrt{7}}{9 + 5\ln(2)}$$

(73) Calculate the following limits.
  (a) $\lim_{x\to 4} 3x - 7$
  (b) $\lim_{x\to 5} \dfrac{3x + 11}{4x - 7}$

(74) Use the definition $f'(c) = \lim_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$ of a derivative to show that if $f(x) = x^2 + 3x - 1$ then $f'(5) = 13$.

(75) Find $\dfrac{d}{dx}(x^8 - 12x^6 + 9x^2 + 3x - 7)$.

(76) Consider the function f defined as follows.

$$f(x) = \begin{cases} x^2 & \text{if } x < 0, \\ 2x + 1 & \text{if } x \geq 0. \end{cases}$$

Evaluate the following.
  (a) $f(-3)$
  (b) $f(-2)$
  (c) $f(0)$
  (d) $f(2)$
  (e) $f(3)$

(77) Using the fact that the graph of a function f is $\{(x, y) : y = f(x)\}$, sketch the graph of the function f defined completely by the following table of values.

x	$f(x)$
-3	5
-2	6
-1	2
0	4
1	-1
2	3

(78) Sketch the graph of the equation $y - 3 = (x + 2)^2$.

(79) Sketch the graph of the exponential function $y = 4^x$.

(80) Use your calculator to evaluate each of the following.
  (a) $2^{3.5}$
  (b) $f(-2)$, where $f(x) = 2 \cdot 3^x$.

(81) Evaluate $\dfrac{2.7 + 3 \cdot 5^{2.1}}{7 \cdot e^{3.2} - 5}$ with your calculator. Write down the brand and model of your calculator, indicate whether or not it is a graphing calculator, and write down *exactly* what sequence of keys you pressed to get your answer.

(82) Assume that you have placed \$527 in a bank account paying interest at an annual rate of 4.7%.
  (a) What will the balance be after 5 years if interest is compounded monthly?
  (b) What will the balance be after 5 years if interest is compounded continuously?

(83) The balance in a bank account is given by the formula $P = 2475(1 + \frac{.04}{4})^{4t}$, where t is the amount of time the money is left in the account and P is the balance.
(a) What is the annual interest rate?
(b) How frequently is interest compounded?
(c) What will the balance be after 25 years?

(84) The balance in a bank account is given by the formula $P = 1732e^{.04t}$, where t is the amount of time the money is left in the account and P is the balance.
(a) What is the annual interest rate?
(b) How frequently is interest compounded?
(c) What will the balance be after 21 years?

(85) You have the choice of putting your money either in an account paying interest at an annual rate of 4.9%, compounded continuously, or 5%, compounded quarterly. Which is the better option and why?

(86) Define, in relatively plain language, the meaning of $\log_b a$. *Note: Saying that it is the logarithm of a to the base b is insufficient.*

(87) Solve the equation $\ln(2x + 1) = 3$ for x.

(88) Evaluate $\dfrac{10 + \ln(5.3^2 + 3.1)}{e^{2.7} + 1.4}$ with your calculator. Complete the evaluation without writing down (or using in any way) any intermediate results. Write down the brand and model of your calculator, indicate whether or not it is a graphing calculator, and write down *exactly* what sequence of keys you pressed to get your answer.

(89) Evaluate $\lim_{x\to 3} 2x - 1$.

(90) Evaluate $\lim_{x\to 2} \dfrac{x^2 + 3x - 10}{x - 2}$.

(91) Suppose that the function $s = 5t^2 + 3t$ measures the distance a rolling stone travels down a hill, where s represents the distance, measured in feet, and t measures the time it has been rolling, measured in seconds.
(a) Find the stone's average speed over the time interval $2 \le t \le 5$.
(b) Find the instantaneous speed of the stone when $t = 4$.

(92) Find $\dfrac{d}{dx}(x^3 - 5x^2 + 7x + 3)$.

(93) Let $f(x) = (x^3 - 5x^2 + 7x + 3) \cdot (5x - 1)$. Find $f'(x)$.

(94) Let $f(x) = \dfrac{x^3 - 5x^2 + 7x + 3}{5x - 1}$. Find $f'(x)$.

(95) Find the slope of the line which is tangent to the graph of $y = x^2 - x$ at the point $(3, 6)$. *Bonus: Explain why the point $(3, 6)$ is on the graph.*

(96) Let $f(x) = x^2 - 12x + 5$. Determine the set of values for which the derivative of $f(x)$ is positive.

(97) Calculate $\lim_{x\to 4} \dfrac{x^2 + 2x - 24}{x - 4}$.

(98) Let $g(x) = (x^2 + 6x + 3)(x^2 - 5)$. Find $g'(x)$.

(99) Let $f(t) = \sqrt{4t + 9}$. Find $f'(t)$.

(100) Suppose the total cost of producing x units of a particular commodity is $C(x) = \dfrac{x^3+3x}{x^2+1}$. What is the marginal cost?

(101) Consider a function $f(x)$ with the following properties.
- (a) f is increasing for $x < -3$ and for $x > 5$.
- (b) f is decreasing for $-3 < x < 5$.
- (c) The graph is concave up for $x > 2$.
- (d) The graph is concave down for $x < 2$.
- (e) $f(-3) = 10$, $f(2) = 7$, $f(5) = 2$.

Sketch the graph of f and identify all extrema and points of inflection.

(102) Consider the function $y = x^4 - 4x^3$.
- (a) Find y' and y''.
- (b) Factor y' and y'' completely.
- (c) Analyze the monotonicity and concavity of the function and its graph.
- (d) Sketch the graph.

(103) A farmer wishes to enclose a rectangular field of area 800 square feet using an existing wall as one of its sides. The cost of the fence for the other three sides is \$7 per linear foot. Find the dimensions of the rectangular field that minimizes the cost of the fence.

(104) Find $\int_1^3 x^2 + 6x - 2\,dx$.

(105) Sketch the plane region bounded by the curve $y = x^4$, the x-axis and the line $x = 4$ and find its area.

(106) Suppose the speed of a car making a jackrabbit start from a red light is given by $v = 2t + 10$ for $0 \le t \le 5$, where v represents the car's speed in feet per second and t represents the time, in seconds, measured from the moment the light turns green. How far does the car go in those five seconds?

(107) $y = x^2$

(108) $y = (x-3)^2$

(109) $y + 2 = x^2$

(110) $y + 2 = (x-3)^2$

(111) $\lim_{x\to 3} \dfrac{x^2+2x-5}{5x-13}$

(112) $\lim_{h\to 0} \dfrac{(3+h)^2-9}{h}$

(113) Consider a function $y = f(x)$ for which you have the following information.
- a. $f(x)$ is increasing for $x < -3$ and for $x > 4$.
- b. $f(x)$ is decreasing for $-3 < x < 4$.
- c. The graph of $y = f(x)$ is concave up for $x > 1$.
- d. The graph of $y = f(x)$ is concave down for $x < 1$.
- e. $f(-3) = 5$, $f(1) = 2$ and $f(4) = 0$.
- (a) Sketch the graph of $y = f(x)$.
- (b) At what point(s) does $f(x)$ attain a maximum?
- (c) At what point(s) does $f(x)$ attain a minimum?
- (d) At what point(s) does the graph of $f(x)$ have a point of inflection?

(114) Calculate $\lim_{x\to 36} \sqrt{x}$.

(115) Calculate $\lim_{x\to 5} \dfrac{x^2+x-30}{x-5}$.

(116) Calculate $\lim_{x\to 64} \dfrac{\sqrt{x}-8}{x-64}$.

(117) Calculate $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$, where $f(x) = 5x+3$.

(118) Suppose the distance travelled by an object is given by the formula $s = t^3+5t^2+3t$, where s is the distance, measured in feet, and t is the time, measured in seconds. Find the average speed of the object over the time interval $2 \le t \le 4$.

(119) Let $f(x) = 3x^5$. Find a formula for $f'(x)$ and find the value of $f'(2)$.

(120) Let $f(x) = x^8 - 5x^6 + 8x^3 - 7x + 3$. Find a formula for $f'(x)$.

(121) Let $f(x) = 3x\sqrt{1+x^2}$. Use the fact that $\dfrac{d}{dx}\left(\sqrt{1+x^2}\right) = \dfrac{x}{\sqrt{1+x^2}}$ to find a formula for $f'(x)$.

(122) Let $f(x) = \dfrac{x^2}{x^4+3x+20}$. Find a formula for $f'(x)$.

(123) Consider the curve $y = x^2 - 6x + 15$ and the line tangent to the curve at the point (3,6). Find the slope and equation of the tangent line.

(124) Calculate the derivatives of each of the following functions.
  (a) $f(x) = x^5 - 3x^4 + x^2 + 11x - 5$
  (b) $g(t) = (t^2+t+1)(t^3-t^2+t-1)$
  (c) $h(x) = \dfrac{x^2+x+1}{x^3-x^2+x-1}$
  (d) $f(x) = (x^4+7x-9)^{10}$
  (e) $g(x) = \sqrt{x^{10}+1}$
  (f) $h(x) = \dfrac{x}{\sqrt{x^2+1}}$

(125) Suppose revenue is given by the formula $R(x) = 300x - x^2$, where x represents demand and $R(x)$ represents revenue.
  (a) Find a formula for the marginal revenue.
  (b) For what demand is the marginal revenue equal to 0?
  (c) (Extra Credit) Find a formula for the demand function $p = D(x)$, where p represents price.

(126) Suppose $x = 500 - 10p$, where p represents price and x represents demand.
  (a) Find a formula for elasticity $\eta = E$.
  (b) For what values of p is $\eta = 1$?
  (c) For what values of p is the demand elastic?
  (d) For what values of p is the demand inelastic?

(127) Consider a demand function $p = 70 - x$, where x represents demand and p represents price.
  (a) Obtain a formula for the revenue function $R(x)$.
  (b) Obtain a formula for the marginal revenue.
  (c) Obtain a formula for the elasticity in terms of the demand x.

(128) \$250 is placed in an account paying interest at an annual rate of 6%, compounded continuously. What will the balance be after 5 years? *Give both an exact answer and a decimal approximation to the nearest cent. For extra credit, find how long it will take for the balance to quadruple.*

(129) Suppose the distance travelled by an object thrown down off a bridge is given by the formula $s = 16t^2 + 64t$, where s is the distance fallen, measured in feet, and t is the length of time since the object is released, measured in seconds.
  (a) How far will the object have fallen after 5 seconds?
  (b) What is the average speed of the object during the first five seconds after it's thrown down?
  (c) Obtain a formula for the instantaneous speed v of the object in terms of t.
  (d) How fast is the object falling after 5 seconds?

(130) Consider the graph of the function $y = \dfrac{x}{x-3}$ along with the line tangent to the graph at the point $(6, 2)$.
  (a) Find the slope of the tangent line.
  (b) Find an equation for the tangent line.

(131) Calculate the following limits.
  (a) $\lim_{x\to 5} \dfrac{x^2 - 3x - 10}{x - 5}$.
  (b) $\lim_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$, where $f(x) = 1/x$. *Extra Credit: Calculate the limit two completely different ways.*

(132) Calculate the derivatives of the following functions.
  (a) $f(x) = x^2/(x^3 - 5x + 2)$
  (b) $y = \sqrt{9x^2 + 4}$

(133) The distance a ball rolls down a 28 foot long inclined plane is given by the formula $s = 5t^2 + 4t$, where s represents distance, measured in feet, and t represents time, measured in seconds from the instant the ball is released.
  (a) How long does it take for the ball to roll down the plane?
  (b) What is the average speed of the ball during the first second it is rolling?
  (c) What is the instantaneous speed of the ball after 1 second?

(134) Consider the curve $y = \sqrt{x^2 + 16}$.
  (a) Find the slope of the line tangent to the curve at the point $(3, 5)$.
  (b) Find an equation of that tangent line.

(135) Consider the demand function $p = 40 - 0.1x$, where x represents demand and p represents price.
  (a) Find a formula for the revenue $R(x)$ in terms of the demand x.
  (b) Find the marginal revenue.
  (c) For what value of the demand x is the marginal revenue equal to 0?
  (d) Find a formula for the elasticity of demand η.
  (e) For what values of the demand x is the demand elastic?

(136) Consider the function $f(x) = x^3 - 9x^2 + 24x - 10$.
  (a) Find $f'(x)$ and $f''(x)$ and factor both as much as possible.

(b) Determine where $f'(x) = 0$ and where $f''(x) = 0$.
(c) Analyze the monotonicity of f. *In other words, determine the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.*
(d) Find and identify all relative extrema (i.e. maxima and minima) for f.
(e) Analyze the concavity of f. *In other words, determine the intervals on which the graph of $f(x)$ is concave up and the intervals on which the graph of $f(x)$ is concave down.*
(f) Find and identify all points of inflection for f.
(g) Sketch the graph of f.

(137) A theater owner charges \$6 per ticket and sells 300 tickets. By checking other theaters, she decides that for every dollar she raises ticket prices, she will lose ten customers. What should she charge in order to maximize her revenue?

(138) Find the following indefinite integrals.
(a) $\int 6x^2 - 3x + 2\,dx$
(b) $\int \dfrac{x}{\sqrt{x^2+4}}\,dx$

(139) Evaluate $\int_2^4 x^3\,dx$.

(140) Find the area of the plane region above the $x-$axis, below the curve $y = x + \dfrac{1}{x}$ and between the lines $x = 1$ and $x = 4$.

(141) Evaluate $\lim_{x\to 3}(x^2 - 2x + 4)$.

(142) Evaluate $\lim_{x\to 4} \dfrac{x^2 - 2x + 8}{x - 4}$.

(143) Suppose a demand function $p = D(x)$ is given by the formula $p = 50 - x$, where x represents demand, in thousands of units, and p represents the price, in dollars.
(a) Obtain a formula for the revenue $R = R(x)$ in terms of demand x.
(b) Obtain a formula for the marginal revenue.
(c) For what demand x is the marginal revenue equal to 0?
(d) Find a formula for the elasticity of demand, which is sometimes denoted by η and sometimes by E.
(e) For what value of the demand x is the elasticity of demand equal to 1?

(144) Let $f(x) = x^2 - 4x + 7$. Evaluate $f(0)$, $f(3)$, $f(-3)$ and $f(1.5)$.

(145) Calculate $\lim_{x\to 36} \sqrt{x}$.

(146) Calculate $\lim_{x\to 5} \dfrac{x^2 + x - 30}{x - 5}$.

(147) Find the balance after 10 years if \$350 is placed in an account paying interest at an annual rate of 6% compounded monthly.

(148) Use the Bisection Method to find an interval of width no greater than 0.05 which contains a solution to the equation $x + \ln x = 10$.

(149) Suppose the distance travelled by an object is given by the formula $s = t^3 + 5t^2 + 3t$, where s is the distance, measured in feet, and t is the time, measured in seconds. Find the average speed of the object over the time interval $2 \le t \le 5$.

(150) Consider the same object as in the previous question. Find its instantaneous speed when $t = 3$.

(151) Let $f(x) = 3x^5$. Find a formula for $f'(x)$ and find the value of $f'(2)$.
(152) Briefly discuss two uses of derivatives.
(153) (a) Explain, in plain language, the meaning of the term *marginal cost.*
(b) Define the term *marginal cost.*
(154) Find $\dfrac{d}{dx}(x^8 - 3x^5 + 17)$.
(155) Find $\dfrac{d}{dx}(x^3 \ln x)$.
(156) Find $\dfrac{d}{dx}\left(\sqrt{x^4 + 2x^2 + 1}\right)$.
(157) Find $\dfrac{d}{dx}(x^4 \ln(x^2 + 1))$.
(158) Find the slope of the tangent line to the graph of $y = x^2 - 3x + 1$ at the point $(4, 5)$.

For the next two questions, suppose an object is travelling in a straight line and its distance from its starting point is given by the formula $s = 10t - t^2$, where t represents time, measured in seconds, and s represents distance, measured in feet.

(159) Find a formula for its velocity v.
(160) Find its speed when $t = 2$.

For the next three questions, assume the demand function is given by the formula $p = 100 - 2x$. As usual, x represents demand and p represents price.

(161) Find a formula for the revenue function $R(x)$.
(162) Find a formula for the marginal revenue.
(163) Find a formula for the elasticity of demand η.

For the next two questions, consider the function $f(x) = (x - 5)e^x$. You may use the fact that e^x is always positive.

(164) Analyze the monotonicity of the function f.
(165) Analyze the concavity of the graph of the function f.
(166) Use your calculator to calculate $\dfrac{5.2 \cdot 3.1^{2.7 \cdot 1.03 - 1} - 2.1 \cdot 1.01^{2.3}}{1.23 + 3.5 \ln(3)}$ without doing any mental calculations or writing down or entering any partial calculations. List the sequence of keys you pressed.
(167) Determine which of the following points are on the graph of the function f defined by the formula $f(x) = x^2 - 3x + 2$. Justify each answer.

(a) $(3, 2)$
(b) $(3, 5)$
(c) $(-5, 42)$

(168) Suppose the distance a car travels from the moment the driver steps on the brake is given by the formula $s = 40t - t^2$, where t represents the time, measured in seconds, since the driver stepped on the brake and s represents the distance travelled, measured in feet. What is the average speed of the car during the time interval $2 \le t \le 4$?
(169) Calculate the following derivative: $\dfrac{d}{dx}(x^5 - 9x^4 - 7x^3 + 5x^2 + 2 + 3e^x)$.
(170) Calculate the following derivative: $\dfrac{d}{dx}(x^2 \ln x)$.

(171) Calculate: $\dfrac{2.35 \cdot 8.9^{1.357+5.18/4.3} - 9.728^2}{2.7\sqrt{5.1} - \ln 7.3}$.