Mathematics 106 SOLUTIONS
Professor Alan H. Stein
Due Monday, July 23 This problem set will be graded on the basis of 100 points but will be worth 50 points.

1. Find an equation for the line with x-intercept 20 and y-intercept 5. Sketch its graph.

   Solution: It goes through $(20, 0)$ and $(0, 5)$, so it has slope $\frac{5-0}{0-20} = -\frac{1}{4}$. Its equation may be written as $y - 0 = -\frac{1}{4}(x - 20)$ or $y - 5 = -\frac{1}{4}(x - 0)$. Its graph may be drawn by simply connecting the two intercepts with a straight line.

2. Factor $x^3 + 3x^2 - 34x - 120$ completely.

   Solution: Looking at the divisors of 120, we see -4 is a zero of the polynomial, so $x - (-4) = x + 4$ is a factor. Factoring $x + 4$ out (using long division, synthetic division or some other method), we get $x^3 + 3x^2 - 34x - 120 = (x + 4)(x^2 - x - 30)$.

   The second factor may be further factored at sight or by trial and error to get $x^3 + 3x^2 - 34x - 120 = (x + 4)(x + 5)(x - 6)$.

3. For what values of x is $\dfrac{x(x+6)}{(x-2)^3}$ negative?

   Solution: We note the numerator is 0 when $x = 0$ and when $x = -6$ and the denominator is 0 when $x = 2$.

   When $x > 2$, all the factors are positive so the quotient is positive.

   When $0 < x < 2$, $x - 2 < 0$, so $(x - 2)^3 < 0$, but the other factors are positive so the quotient is negative.

   When $-6 < x < 0$, $x < 0$, $(x - 2)^3$ is still negative, while $x + 6 > 0$, so the quotient is positive.

   When $x < -6$, $x + 6 < 0$ and x and $(x - 2)^3$ remain negative, so the quotient is negative.

   We conclude $\dfrac{x(x+6)}{(x-2)^3}$ is negative when $x < -6$ and when $0 < x < 2$. We may describe the set for which $\dfrac{x(x+6)}{(x-2)^3}$ is negative as $\{x|x < -6 \text{ or } 0 < x < 2\} = (-\infty, -6) \cup (0, 2)$.

4. Calculate $\lim_{x\to5} \dfrac{x^2 + 4x - 45}{x - 5}$.

   Solution: $\lim_{x\to5} \dfrac{x^2 + 4x - 45}{x - 5} = \lim_{x\to5} \dfrac{(x + 9)(x - 5)}{x - 5} = \lim_{x\to5}(x + 9) = 14.$

5. The distance s (in feet) traversed along a straight path by an object by time t (in seconds) is given by the formula $s = 8t^3 + 5t^2 + 2t$.

   (a) Find its average speed during the time interval $2 \le t \le 4$.

   Solution: Its average speed is $\dfrac{s|_{t=4} - s|_{t=2}}{2} = \dfrac{600 - 88}{2} = \dfrac{512}{2} = 256$ feet per second.

   (b) Find its instantaneous speed when $t = 3$.

   Solution: Its instantaneous speed is $s'|_{t=3}$. $s' = 24t^2 + 10t + 2$, so its instantaneous speed is 248 feet per second.

6. Let $f(x) = x^3 + 5x$. Use the definition of a derivative to find $f'(x)$.

   Solution: $f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{z^3+5z-(x^3+5x)}{z-x} = \lim_{z\to x} \frac{z^3-x^3+5z-5x}{z-x} = \lim_{z\to x} \frac{(z-x)(z^2+zx+x^2)+5(z-x)}{z-x} = \lim_{z\to x} \frac{(z-x)(z^2+zx+x^2+5)}{z-x} = \lim_{z\to x}(z^2 + zx + x^2 + 5) = 3x^2 + 5$.

7. Calculate $\dfrac{d}{dx}(x^5 + 9x^2 - 2\tan x)$.

   Solution: $\dfrac{d}{dx}(x^5 + 9x^2 - 2\tan x) = 5x^4 + 18x - 2\sec^2 x$.

8. Calculate $\dfrac{d}{dx}((x^2 + 4x - 1)e^x)$.

   Solution: $\dfrac{d}{dx}((x^2 + 4x - 1)e^x) = (x^2 + 4x - 1)\dfrac{d}{dx}(e^x) + e^x\dfrac{d}{dx}(x^2 + 4x - 1) = (x^2 + 4x - 1)e^x + e^x(2x + 4) = e^x(x^2 + 6x + 3)$.

9. Calculate $\dfrac{d}{dx}\left(\dfrac{x^3 + 5x - \cos x}{3 + \sin x}\right)$.

   Solution: $\dfrac{d}{dx}\left(\dfrac{x^3 + 5x - \cos x}{3 + \sin x}\right) =$
   $$\frac{(3 + \sin x)\frac{d}{dx}(x^3 + 5x - \cos x) - (x^3 + 5x - \cos x)\frac{d}{dx}(3 + \sin x)}{(3 + \sin x)^2} =$$
   $$\frac{(3 + \sin x)(3x^2 + 5 + \sin x) - (x^3 + 5x - \cos x)(\cos x)}{(3 + \sin x)^2}.$$

10. Write down a strategy for calculating derivatives.

    Solution: See notes online.