Mathematics 105 | SOLUTIONS
Professor Alan H. Stein
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1. Use Gaussian Elimination to solve the following system of equations.

$$\begin{aligned} 5x + 3y &= 25 \\ 3x + 4y &= 4 \end{aligned}$$

Solution: $\begin{pmatrix} 5 & 3 & 25 \\ 3 & 4 & 4 \end{pmatrix}$ $\qquad R_1 \leftarrow R_1/5$: $\Big(1 \quad 3$

3. Consider the following linear programming problem:

   Constraints:

$$\begin{aligned} x + y &\le 80 \\ 3x &\le 90 \end{aligned}$$

   Non-negativity conditions: $x \ge 0$, $y \ge 0$

   The objective function $M = 5x + 3y$ is to be maximized.

   Set up the initial simplex tableau and use the Simplex Method to maximize the objective function.

   Solution: The initial simplex tableau is $\begin{pmatrix} 1 & 1 & 1 & 0 & 80 \\ 3 & 0 & 0 & 1 & 90 \\ -5 & -3 & 0 & 0 & 0 \end{pmatrix}$

   We now use the Simplex Method:

   $R_2 \leftarrow R_2/3$: $\begin{pmatrix} 1 & 1 & 1 & 0 & 80 \\ 1 & 0 & 0 & 1/3 & 30 \\ -5 & -3 & 0 & 0 & 0 \end{pmatrix}$

   $R_1 \leftarrow R_1 - R_2$, $R_3 \leftarrow R_3 + 5R_2$: $\begin{pmatrix} 0 & 1 & 1 & -1/3 & 50 \\ 1 & 0 & 0 & 1/3 & 30 \\ 0 & -3 & 0 & 5/3 & 150 \end{pmatrix}$

   $R_3 \leftarrow R_3 + 3R_1$: $\begin{pmatrix} 0 & 1 & 1 & -1/3 & 50 \\ 1 & 0 & 0 & 1/3 & 30 \\ 0 & 0 & 3 & 2/3 & 300 \end{pmatrix}$

   We conclude the objective function is maximized when $x = 30$, $y = 50$ and that its maximum value is 300.

4. What will the balance be after 25 years if \$750 is placed in an account paying interest at an annual rate of 4.5%, compounded monthly?

   Solution: $750(1 + 0.045/12)^{25 \cdot 12} = 750(1 + 0.045/12)^{300}$.

5. What will the balance be after 25 years if \$750 is deposited monthly in an account paying interest at an annual rate of 4.5%, compounded monthly?

   Solution: $750 s_{\overline{300}|0.045/12} = 750 \cdot \dfrac{(1 + 0.045/12)^{300} - 1}{0.045/12}$.

6. What is the monthly payment on a \$6,000 loan being repaid over five years if the annual interest rate is 8%?

   Solution: $\dfrac{6,000}{a_{\overline{60}|0.08/12}} = \dfrac{6,000}{\frac{1-(1+0.08/12)^{-60}}{0.08/12}} = \dfrac{6,000 \cdot 0.08}{0.08(1 - (1 + 0.08/12)^{-60})}$

7. Calculate $P(6, 4)$.

   Solution: $P(6, 4) = 6 \cdot 5 \cdot 4 \cdot 3 = 360$.

8. Calculate $C(6,4)$.

   Solution: $C(6,4) = \dfrac{6!}{4!2!} = \dfrac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\cdot2\cdot1} = \dfrac{6\cdot5}{2} = 15.$

9. How many license plates are possible if each license plate consists of two letters, followed by three digits and ending with either a letter or a digit?

   Solution: $26\cdot26\cdot10\cdot10\cdot10\cdot36 = 10^3\cdot26^2\cdot36.$

10. Quality control chooses five phones to test from a production run of 150 phones. How many ways can this be done?

    Solution: $C(150,5) = \dfrac{150!}{5!145!}.$

11. In how many ways can six commercials be scheduled into the six slots available during a half hour television show?

    Solution: $P(6,6) = 6! = 720.$

12. A bridge hand consists of thirteen cards. How many different bridge hands are there?

    Solution: $C(52,13) = \dfrac{52!}{13!39!}.$

13. How many different bridge hands contain exactly six hearts, including the ace of hearts?

    Solution: $C(12,5)C(39,7)$

14. What is the probability of being dealt a bridge hand containing exactly six hearts, including the ace of hearts?

    Solution: $\dfrac{C(12,5)C(39,7)}{C(52,13)}$

15. Find the mathematical expectation $E(X)$ for a random variable X with the following probability distribution:

k	Pr(X=k)
1	0.3
2	0.5
4	0.2

Solution: $E(X) = 1\cdot0.3 + 2\cdot0.5 + 4\cdot0.2 = 0.3 + 1 + 0.8 = 2.1.$