Mathematics 105 SOLUTIONS
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1. List the three elementary row operations on matrices.

   Solution:

   - Interchange two rows.
   - Multiply (or divide) a row by a non-zero constant.
   - Add (or subtract) a multiple of one row to (or from) another row.

2. Consider the following arithmetical expression:
   $\dfrac{5.1 \cdot 2.3^{1.7+0.4\cdot 1.3} + 8.2 \ln 2.9}{\frac{5.3}{1.7} - 2.1 \cdot 0.043}$.
   Use your calculator, abiding by the restrictions given for the similar question in class, to evaluate the expression to the full precision available. List the sequence of keys you pressed. For example, if you were calculating $5 + 3 \ln 8.3$, you might list $5 + 3 \times \ln(8.3)$ enter.

   Solution: $(5.1 \times 2.3 \wedge (1.7 + 0.4 \times 1.3) + 8.2 \times \ln(2.9)) \div (5.3 \div 1.7 - 2.1 \times 0.043)$ ENTER

   The result is 13.5878627545.

3. Sketch the feasible set for the following system of inequalities:

   $2x + y \leq 8$
   $3x - y \geq 2$
   $x \geq 0$, $y \geq 0$.

   Solution: The line $2x + y = 8$ has x-intercept 4 and y-intercept 8. The inequality is satisfied by the points below the line, so we shade in the points above the line.

   The line $3x - y = 2$ has x-intercept $\frac{2}{3}$ and y-intercept -2. The inequality is satisfied by the points to the right of the line, so we shade in the points to the left.

   We also shade in the points to the left of the y-axis and the points below the x-axis, leaving a triangle in the first quadrant as the feasible set.

   The points $(\frac{2}{3}, 0)$ and $(4, 0)$ along the x-axis form two of the vertices. To find the third vertex, we see where the lines intersect.

   We may use elimination to solve the equations $2x + y = 8$ and $3x - y = 2$ simultaneously. "Adding," we get $5x = 10$, $x = 2$. Plugging that into either equation yields $y = 4$, so the third vertex is $(2, 4)$.

   Thus the feasible set is the interior and boundary of the triangle with vertices $(\frac{2}{3}, 0)$, $(4, 0)$, $(2, 4)$.

4. Consider supply and demand functions $S(x) = 2 \cdot 10^{-6}x$ and $D(x) = 30 - 5 \cdot 10^{-7}x$. Using the notation used in class, sketch their graphs and find the equilibrium price and demand.

   Solution: It's easiest to use "Substitution" to solve the equations $p = 2 \cdot 10^{-6}x$, $p = 30 - 5 \cdot 10^{-7}x$ simultaneously.

   $2 \cdot 10^{-6}x = 30 - 5 \cdot 10^{-7}x$, $2 \cdot 10^{-6}x + 5 \cdot 10^{-7}x = 30$, $20 \cdot 10^{-7}x + 5 \cdot 10^{-7}x = 30$, $25 \cdot 10^{-7}x = 30$, $x = \dfrac{30}{25 \cdot 10^{-7}} = \dfrac{30 \cdot 10^{7}}{25} = \dfrac{300 \cdot 10^{6}}{25} = 12 \cdot 10^{6} = 12,000,000.$

   We then get $p = 2 \cdot 10^{-6} \cdot 12 \cdot 10^{6} = 24$.

   We thus have an equilibrium price of 24 and an equilibrium demand of 12 million.

5. Write down the augmented matrix for the system

$$\begin{aligned} 5x - 3y + 8z &= 83 \\ 4x + 7y - 9z &= 41 \\ -2x + 5z &= 53. \end{aligned}$$

   Solution:
$$\begin{pmatrix} 5 & -3 & 8 & 83 \\ 4 & 7 & -9 & 41 \\ -2 & 0 & 5 & 53 \end{pmatrix}$$

6. Pivot about the entry in the second row, third column of the matrix
$$\begin{pmatrix} 5 & 3 & 2 & 4 \\ 2 & 4 & 2 & 8 \\ 2 & -2 & 5 & 7 \end{pmatrix}$$

   Solution:
$$R_2 \leftarrow \frac{R_2}{2} : \begin{pmatrix} 5 & 3 & 2 & 4 \\ 1 & 2 & 1 & 4 \\ 2 & -2 & 5 & 7 \end{pmatrix}, \; R_1 \leftarrow R_1 - 2R_2 : \begin{pmatrix} 3 & -1 & 0 & -4 \\ 1 & 2 & 1 & 4 \\ 2 & -2 & 5 & 7 \end{pmatrix}$$
$$R_3 \leftarrow R_3 - 5R_2 : \begin{pmatrix} 3 & -1 & 0 & -4 \\ 1 & 2 & 1 & 4 \\ -3 & -12 & 0 & -13 \end{pmatrix}$$

7. Use Gaussian Elimination on the following matrix.

$$\begin{pmatrix} 1 & 3 & 5 & 2 \\ 2 & 3 & 4 & 4 \\ 3 & 7 & 9 & 1 \end{pmatrix}$$

Solution: We start by pivoting about the first row, first column. Since there's already a 1 there, we merely need to get 0's elsewhere in the first column.

$$R_2 \leftarrow R_2 - 2R_1 : \begin{pmatrix} 1 & 3 & 5 & 2 \\ 0 & -3 & -6 & 0 \\ 3 & 7 & 9 & 1 \end{pmatrix}, \; R_3 \leftarrow R_3 - 3R_1 : \begin{pmatrix} 1 & 3 & 5 & 2 \\ 0 & -3 & -6 & 0 \\ 0 & -2 & -6 & -5 \end{pmatrix}.$$

We now pivot about the second row, second column. We start by getting a 1 in the pivot entry by dividing the second row by -3.

$$R_2 \leftarrow \frac{R_2}{-3} : \begin{pmatrix} 1 & 3 & 5 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & -6 & -5 \end{pmatrix}.$$

We now get 0's elsewhere in the second column.

$$R_1 \leftarrow R_1 - 3R_2 : \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & -6 & -5 \end{pmatrix}, \; R_3 \leftarrow R_3 + 2R_2 : \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & -5 \end{pmatrix}.$$

We now pivot about the third row, third column. We start by getting a 1 in the pivot entry by dividing the third row by -2.

$$R_3 \leftarrow \frac{R_3}{-2} : \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & \frac{5}{2} \end{pmatrix}.$$

We now get 0's elsewhere in the pivot column:

$$R_1 \leftarrow R_1 + R_3 : \begin{pmatrix} 1 & 0 & 0 & \frac{9}{2} \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & \frac{5}{2} \end{pmatrix}, \; R_2 \leftarrow R_2 - 2R_3 : \begin{pmatrix} 1 & 0 & 0 & \frac{9}{2} \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & \frac{5}{2} \end{pmatrix}.$$

8. After using Gaussian Elimination to solve a system of equations, you obtain the following matrix. Interpret the solution set.

$$\begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Solution: If the variables are x, y, z, the rows correspond to the equations $x + 5y = 0$, $z = 0$ and $0 = 0$. We can pick any value we want for the "free" variable y, let $x = -5y$ and $z = 0$.

We may describe the solution set as the set of values for x, y, and z for which $x = -5y$ and $z = 0$.

9. After using Gaussian Elimination to solve a system of equations, you obtain the following matrix. Interpret the solution set.

$$\begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution: If the variables are x, y, z, the rows correspond to the equations $x + 5y = 0$, $z = 0$ and $0 = 1$. Since the last clearly has no solution, neither does the system and the solution set is empty.