Mathematics 116 SOLUTIONS
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1. Using relatively plain language, define each of the following terms.
   (a) Present value
   (b) Future value
   (c) Annuity

   Solution: See notes.

2. Suppose \$200 is placed in an account paying interest at an annual rate of 3%, compounded quarterly.
   (a) What is the interest rate applied each quarter?
   Solution: $\frac{1}{4}$ of 3%, or $\frac{1}{4} \cdot 0.03 = 0.0075 = 0.75\%$
   (b) What is the interest credited at the end of the first quarter?
   Solution: $0.0075 \cdot 200 = 1.5$, so the bank will credit \$1.50 interest.
   (c) What is the balance after interest is credited the first time?
   Solution: \$201.50.

3. \$250 is deposited at the end of each month for a period 20 years. If interest is paid at an annual rate of 6%, compounded monthly, how much will be in the account after 20 years?

   Solution: We can use the formula $F = s_{\overline{n}|i}R$ for future value F. In this case, the monthly rent $R = 250$, the number of payments $n = 20 \cdot 12 = 240$ since there are 12 payments a year for 20 years, and the periodic interest rate applied is $\frac{1}{12} \cdot 0.06 = 0.005$, since interest is compounded twelve times per year.

   Thus, the future value is $F = s_{\overline{240}|0.005} \cdot 250$.

   $s_{\overline{240}|0.005} = \frac{(1+0.005)^{240}-1}{0.005}$, so $F = \frac{(1+0.005)^{240}-1}{0.005} \cdot 250 \approx 462.040895162 \cdot 250 \approx 115,510.22379$, so the balance will be \$115,510.22.

4. How much must be deposited at the end of each month into an account which pays interest at an annual rate of 6%, compounded monthly, in order to accumulate one million dollars in a quarter of a century?

   Solution: We may use the same formula, but this time we know the future value $F = 1,000,000$ and want to find the monthly rent R. In this case, the monthly interest rate will be $i = \frac{1}{12} \cdot 6\% = \frac{1}{12} \cdot 0.06 = 0.005$ and the number of payments will be $25 \cdot 12 = 300$, so $1,000,000 = s_{\overline{300}|0.005}R$ or $R = \frac{1,000,000}{s_{\overline{300}|0.005}} = \frac{1,000,000}{\frac{(1+0.005)^{300}-1}{0.005}} = \frac{1,000,000 \cdot 0.005}{1.005^{300}-1} \approx 1,443.01401486$, so the amount deposited each month must be \$1,443.02.

5. You put a significant amount of money into a bank account, paying interest at an annual rate of 3%, compounded quarterly, and then forget about it until a quarter of a century later, at which point you realize you're a millionnaire. How much did you originally place in the account?

   Solution: Letting P_0 be the initial amount deposited, we have, using the compound interest formula, $P_0(1 + 0.03/4)^{25\cdot 4} = 1,000,000$, so $1.0075^{100}P_0 = 1,000,000$ or $P_0 = \dfrac{1,000,000}{1.0075^{100}} \approx 473,690.329599$, so the amount initially placed in the account must have been $\$473,690.33$.

6. You borrow \$15,000 for a car, to be repaid in equal monthly installments over a four year period. The bank charges interest at an annual rate of 8%.

   (a) What is your monthly payment?

   Solution: We may use the formula $P = a_{\overline{n}|i}R$ for the initial value of an annuity, with $a_{\overline{n}|i} = \dfrac{(1+i)^n - 1}{i(1+i)^n} = \dfrac{1-(1+i)^{-n}}{i}$, so $R = \dfrac{P}{a_{\overline{n}|i}} = \dfrac{P}{\frac{1-(1+i)^{-n}}{i}} = \dfrac{Pi}{1-(1+i)^{-n}}$.

   In this case, P is the initial amount of the loan, $15,000$. The monthly interest rate is $i = \dfrac{1}{12}\cdot 0.08 = \dfrac{0.02}{3}$ and the number of monthly payments is $n = 4 \cdot 12 = 48$, so $R = \dfrac{15,000 \cdot \frac{0.02}{3}}{1-(1+.02/3)^{-48}} \approx 366.193834968$, so the monthly payment will be \$366.20.

   (b) What is the total amount of the payments you make to the bank over the course of the loan?

   Solution: $48 \cdot 366.20 = 17,577.6$, so your payments will total $\$17,577.60$.

   (c) How much do you pay in interest to the bank over the course of the loan?

   Solution: The interest paid is the difference between your initial loan amount and your payments, so the interest will total $\$2,577.60$.

7. You take out a \$250,000 mortgage from a bank at an annual rate of 6.3%. Your first monthly payment is $\$1,400$. How much do you still owe the bank after that first payment?

   Solution: Your interest payment will be $\dfrac{1}{12}\cdot 0.063 \cdot 250,000 \approx 1,312.5$, so your interest payment will be $\$1,312.50$. The rest of your payment, \$87.50, will go towards paying down your balance, leaving you with a balance of $\$249,912.50$.

8. Consider the following *word problem*:

   A region with an area of 10 square feet is enclosed by an isoceles triangle with perimeter 14 feet. What are its dimensions.

   (a) List all the variables and unknowns and represent each by a symbol.

   Solution:

   - The base of the triangle. Call it b
   - The height of the triangle. Call it h.
   - The length of the two legs of equal size. Call each x.

   (b) Translate the facts that the region has an area of 10 square feet and the isoceles triangle has a perimeter of 14 feet mathematically, in terms of the variables and unknowns you listed.

   Solution:

   - $\frac{bh}{2} = 10$
   - $b + 2x = 14$