Mathematics 103 Solutions
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July 7, 2004 Final Examination

(1) Consider an election among four candidates: James, Theresa, Marie and Donnie. Suppose 1850 voters cast ballots on which they recorded their preferences as shown in the table below.

Place	570	460	420	400
1st	Marie	James	James	Donnie
2nd	Theresa	Donnie	Donnie	Theresa
3rd	Donnie	Theresa	Marie	Marie
4th	James	Marie	Theresa	James

(a) Who would win under the plurality method?
Solution: James would win with 880 votes to 570 votes for Marie and 400 votes for Donnie.

(b) Who would win if Borda's System was used, with the first choice for each voter receiving 4 points, the second choice receiving 3 points, the third choice receiving 2 points and the last choice receiving 1 point? Show the Borda Count for each candidate.
Solution: The Borda Counts are as follows:
James: $570 \times 1 + 460 \times 4 + 420 \times 4 + 400 \times 1 = 4490$
Theresa: $570 \times 3 + 460 \times 2 + 420 \times 1 + 400 \times 3 = 4250$
Marie: $570 \times 4 + 460 \times 1 + 420 \times 2 + 400 \times 2 = 4380$
Donnie: $570 \times 2 + 460 \times 3 + 420 \times 3 + 400 \times 4 = 5380$
So Donnie will win.

(2) It is 2525 and all the planets in the galaxy have finally signed a peace treaty. Four of the planets (Alanos, Betta, Conii, Dugos, and Ellisium) decide to join forces and form an Intergalactic Federation. The Federation will be ruled by an Intergalactic Congress consisting of 30 seats, and each of the 5 planets will be entitled to a number of seats that is proportional to its population. The population of each planet is:

Planet	Alanos	Betta	Conii	Dugos	Ellisium	Total
Pop (billions)	140	160	125	105	270	800

(a) Determine the *natural divisor*.
Solution: *Note: All population figures are in billions.* Since the total population is 800, the natural divisor is $800/30 = 80/3 \approx 26.6667$.

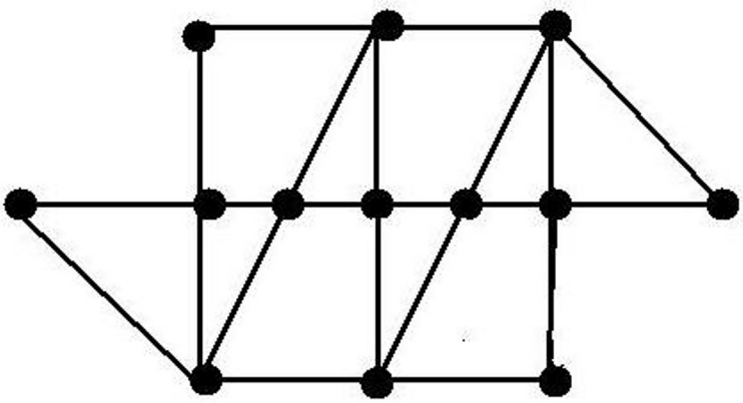
(b) Determine the *natural quota* for each state. *For these and other calculations, round each answer to four places after the decimal point.*
Solution:
Alanos: $140/26.6667 = 5.2500$
Betta: $160/26.6667 = 6.0000$ (rounded from 5.99999250001)
Conii: $125/26.6667 = 4.68749$
Dugos: $105/26.6667 = 3.9375$
Ellisium: $270/26.6667 = 10.1250$

(c) Determine the number of seats each planet would have if its natural quota was rounded down. How many seats short would the Intergalactic Congress be?
Solution:
Alanos: 5
Betta: 6
Conii: 4
Dugos: 3
Ellisium: 10
There would be 28 seats, so the Congress will be 2 seats short.

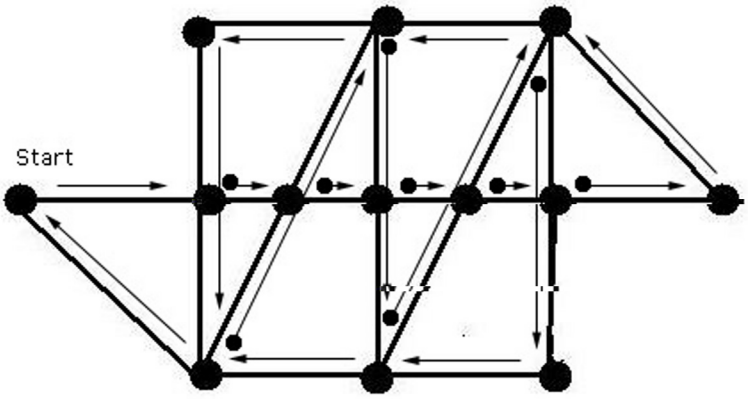
(d) Recall that in *Jefferson's Method*, a modified divisor is obtained so that when the modified quota for each *state* is rounded down, the *house* will have the desired number of seats. Find the *threshold divisor* needed to increase Conii's modified quota to 5 under *Jefferson's Method*.
Solution: $125/5 = 25$

(3) Suppose \$500 is placed in an account paying interest at an annual rate of 3.7%, compounded quarterly. What will the balance be if the account is left undisturbed for 15 years?
Solution: $500(1 + .037/4)^{4 \cdot 15} = 868.75134361$, so the balance will be \$868.75.

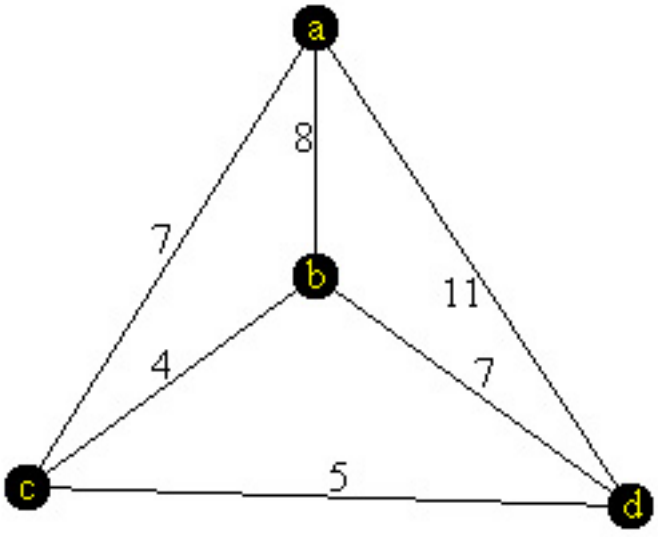
(4) Consider the following graph. Determine whether there is an Euler circuit (a circuit going along each edge exactly once). If so, determine an Euler circuit. If not, explain why.



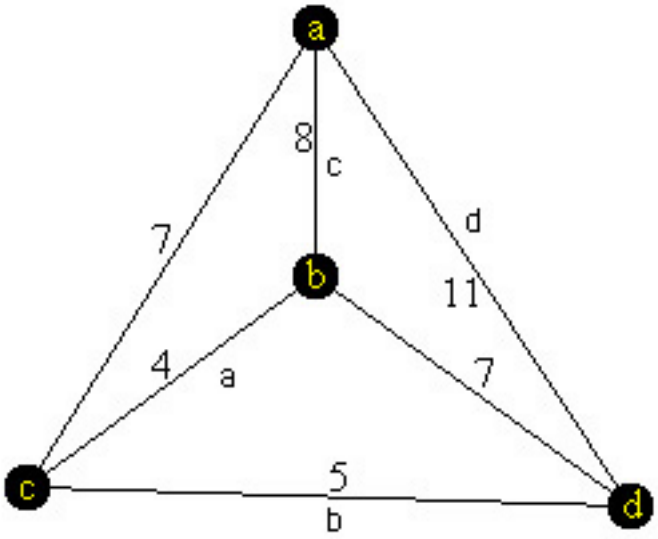
Solution: The following is just one solution. In cases where there is a choice of edges from a vertex, the edge taken first is marked by a small disk.



(5) Consider the following weighted graph. Use the Greedy Algorithm to find an approximate solution to the traveling salesman problem for a circuit starting at vertex D. *Recall we need a circuit going through each vertex exactly once.*



Solution: The Greedy Algorithm gives the path from D to C to B to A to D or the reverse. The diagram below shows the order in which the edges would be determined.



(6) Without using division, determine whether $23,509,851$ is divisible by 3.
Solution: $2+3+5+0+9+8+5+1=33$, which is divisible by 3, so $23,509,851$ must be divisible by 3.

(7) Without using division, determine whether $23,509,851$ is divisible by 6.
Solution: It can't be, since it's odd.

(8) Without using division, determine whether $23,509,851$ is divisible by 9.
Solution: $2+3+5+0+9+8+5+1=33$, which is not divisible by 9, so $23,509,851$ isn't either.

(9) Without using division, determine whether $23,509,851$ is divisible by 11.
Solution: $2-3+5-0+9-8+5-1=9$. Since 9 is not divisible by 11

(10) Calculate $27\times 73+10 \mod 6$.
Solution: $27\times 73+10\equiv 3\times 1+4=7\equiv 1 \mod 6$.

(11) Suppose a pair of dice is tossed. What's the probability the total comes up as 6?
Solution: There are 36 equally likely possible outcomes, of which 5 result in a sum of 6, so the probability is 5/36.

(12) Use the Fundamental Principle of Counting to explain why an ordinary deck of cards contains 52 cards.
Solution: Each card has one of 13 possible face values (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K) along with one of 4 possible suits (clubs, diamonds, hearts, spades), so there are $13\times 4=52$ possibilities.