Mathematics 103 Solutions
Professor Alan H. Stein
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(1) A \$500 initial deposit is made to a bank account paying interest at an annual rate of 4% and the account is left undisturbed for a period of years.
   (a) What will the balance be after ten years if the bank uses simple interest?
   Solution: The amount of interest will be $500 \cdot 0.04 \cdot 10 = 200$, so the balance will be \$700.
   (b) What will the balance be after ten years if the interest is compounded quarterly?
   Solution: $500(1 + .04/4)^{4\cdot 10} = 500 \cdot 1.01^{40} = 744.431866795$, so the balance will be \$744.43.
   (c) What will the balance be after ten years if the interest is compounded continuously?
   Solution: $500e^{.04\cdot 10} = 500e^{0.4}$

(2) Starting at the end of June 2004, deposits of \$600 are made quarterly to an account paying interest at an annual rate of 6%, compounded quarterly. How much will be in the account January 1, 2030?

Solution: We may use the formula $F = D\left(\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right)$ with $D = 600$, $r = .06$, $n = 4$, $t = 25\frac{3}{4}$ to obtain $F = 600\left(\dfrac{\left(1+\frac{.06}{4}\right)^{4\cdot 25\frac{3}{4}}-1}{\frac{.06}{4}}\right) \approx 145379.771709$, so the balance will be $\$145,379.77$.

(3) Suppose monthly deposits are made to an account paying interest at an annual rate of 5%, compounded monthly, for a period of twenty years. How much must each monthly deposit be if the last monthly deposit is to bring the balance up to \$250 thousand?

Solution: We may use the same formula, $F = D\left(\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right)$, with $F = 250000$, $n = 12$, $r = 0.05$, $t = 20$. We get $250000 = D\left(\dfrac{\left(1+\frac{.05}{12}\right)^{12\cdot 20}-1}{\frac{.05}{12}}\right)$. We may proceed as follows.

$D = \dfrac{250000}{\left(\dfrac{\left(1+\frac{.05}{12}\right)^{12\cdot 20}-1}{\frac{.05}{12}}\right)} \approx 608.222680608$. So the amount deposited monthly will need to be \$608.23.

(4) A mortgage in the amount of $\$300,000$ is taken out, to be repaid with equal monthly payments over a period of twenty-five years. The annual interest rate is 6%.

(a) What is the monthly payment?

Solution: We may use the formula $R = \dfrac{P\cdot\frac{r}{n}}{1-\left(1+\frac{r}{n}\right)^{-nt}}$ with $P = 300000$, $n = 12$, $r = 0.06$, $t = 25$, obtaining $R = \dfrac{300000\cdot\frac{.06}{12}}{1-\left(1+\frac{.06}{12}\right)^{-12\cdot 25}} \approx 1932.90420446$.

So the monthly payment will have to be \$1932.91.

(b) What is the total amount of interest paid over the life of the loan?

Solution: The total payments will be $1932.91 \cdot 12 \cdot 25$ = 579873. Since the loan was for \$300000, the amount of interest will be \$279, 873.

(5) \$10, 000 is borrowed at an annual interest rate of 8%. What is the balance owed after four monthly payments of \$80 have been made?
Solution: The following table shows the amount of interest charged for each period, computed by taking the previous month's balance and multiplying by $0.08/12$, along with the balance at the end of the month, obtained by taking the previous month's balance, adding the interest and subtracting the \$80 payment.

Month	Interest	Balance
0		10000
1	66.666666667	9986.6666667
2	66.577777778	9973.2444444
3	66.488296296	9959.7327407
4	66.398218272	9946.130959

So the balance after four payments will be \$9, 946.13.

(6) List all prime numbers smaller than 40.
Solution:
$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$.

(7) Find the unique integers q, r with $0 \leq r < 197$ such that $8569 = q \cdot 197 + r$.
Solution:
$8569/197 \approx 43.4974619289$, so $q = 43$. Then $r = 8569 - q \cdot 197 = 8569 - 43 \cdot 197 = 98$.

(8) Find the least common multiple $\{1500, 90\}$.
Solution: We may factor $1500 = 3 \cdot 500 = 3 \cdot 5 \cdot 100 = 3 \cdot 5 \cdot 10^2 = 3 \cdot 5 \cdot (2 \cdot 5)^2 = 2^2 \cdot 3 \cdot 5^3$ and $90 = 2 \cdot 45 = 2 \cdot 3 \cdot 15 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$.
It follows that $\{1500, 90\} = 2^2 \cdot 3^2 \cdot 5^3 = 4500$.

(9) Find the greatest common divisor $(1500, 90)$.
Solution: Using the same factorizations, we get $(1500, 90) = 2 \cdot 3 \cdot 5 = 30$.
Alternatively, we may use the Euclidean Algorithm to obtain:
$1500 = 16 \cdot 90 + 60$
$90 = 1 \cdot 60 + 30$
$60 = 2 \cdot 30 + 0$
So the greatest common divisor must be 30.

(10) Determine whether 592378531 is divisible by 9 without using division.
Solution: Adding the digits, we get $5 + 9 + 2 + 3 + 7 + 8 + 5 + 3 + 1 = 43$. Repeating, $4 + 3 = 7$. Since 7 is not divisible by 9, neither is 43 and thus neither is 592378531.

(11) Determine whether 592378531 is divisible by 11 without using division.
Solution: $5 - 9 + 0 - 2 + 3 - 7 + 8 - 5 + 3 - 1 = -5$. Since -5 is not divisible by 11, neither is 592378531.