Probability

Definition 1 (Experiment). *An experiment is a process that can be repeated and may result in different outcomes.*

Definition 2 (Event). *An event is a subset of the set of possible outcomes of an experiment.*

The probability of an event may be thought of as the proportion of the times the experiment is performed in which one expects the event to occur.

Notation 1. *If we represent an event by E, we may represent the probability the event occurs by $P(E)$.*

Obvious Probability Formulas

- $0 \leq P(E) \leq 1$
- $P(E) + P(\text{not } E) = 1$
- $P(\text{not } E) = 1 - P(E)$

Equiprobable Probability Spaces

If all the outcomes of an experiment are equally likely, $P(E) =$

$$\frac{\text{number of outcomes that result in the event } E}{\text{total number of possible outcomes}}.$$

It is important to be able to count both the number of possible outcomes and the number of outcomes that result in a given event. The branch of mathematics dealing with counting is called *Combinatorics*.

Odds

If we expect an event to occur b times for every a times it does not occur, we say the odds *against* the event are a to b and the odds *for* the event are b to a.

The probability of the event would be $\dfrac{b}{a+b}$.

If we call the event E, then $P(E) = \dfrac{b}{a+b}$, $1 - P(E) = 1 - \dfrac{b}{a+b} = \dfrac{a}{a+b}$ and thus

$$\frac{a}{b} = \frac{\frac{a}{a+b}}{\frac{b}{a+b}} = \frac{1 - P(E)}{P(E)}.$$

The Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Definition 3 (Mutually Exclusive Events). *Events A and B are mutually exclusive if $P(A \text{ and } B) = 0$.*

If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

Conditional Probability

Definition 4 (Conditional Probability). *The probability that an event B will occur given that event A has occurred is called the conditional probability of B given A and is denoted by $P(B|A)$.*

Notation 2. *Given an event E, we let $n(E)$ represent the number of outcomes that result in the event E.*

If we have an equiprobable space,
$$P(B|A) = \frac{n(A \text{ and } B)}{n(A)}.$$

This suggests that in an equiprobable space, $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$. *This formula holds for all probability spaces.*

The Multiplication Rule

Multiplying both sides of that formula by $P(A)$ results in a useful formula known as *The Multiplication Rule*: $P(A \text{ and } B) = P(A)P(B|A)$.

Independent Events

If the occurrence of one event has no effect on the likelihood of another event, the two events are said to be independent. If the events are A and B, this means $P(B|A) = P(B)$. In this case, the Multiplication Rule reduces to $P(A \text{ and } B) = P(A)P(B)$. This is called the *Multiplication Rule for Independent Events*.

Combinatorics – Counting Techniques

Fundamental Principle of Counting

If there are x_1 ways for one task to be performed and, after that task is performed, there are x_2 ways for a second task to be performed, then there are $x_1 \cdot x_2$ possible ways for the two tasks to be performed in succession.

Variation: Suppose two choices must be made in succession. If there are x_1 alternatives for the first choice and, after that first choice is made, there are x_2 alternatives for the second choice, then there are $x_1 \cdot x_2$ possible ways for the two choices to be made in succession.

The *Fundamental Principle of Counting* generalizes to an arbitrary number of tasks or an arbitrary sequence of choices.

Definition 5 (Permutation). *A permutation is an arrangement or listing of objects.*

We will refer to two different types of permutations: *permutations with replacement* and *permutations without replacement*.

Definition 6 (Permutation With Replacement). *A permutation with replacement is a permutation in which the same item may appear more than once.*

Definition 7 (Permutation Without Replacement). *A permutation without replacement is a permutation in which no item may appear more than once.*

Notation 3. *${}_nP_r$ denotes the number of permutations (without replacement) of r items chosen from a set of size n. We will refer to this as the number of permutations of n objects taken r at a time.*

From the *Fundamental Principle of Counting*, it follows immediately that

$${}_nP_r = n\cdot(n-1)\cdot(n-2)\ldots(n-(r-1)) = \frac{n!}{(n-r)!}.$$

Combinations

Definition 8 (Combination). *A combination is a subset.*

Notation 4. ${}_nC_r$ *denotes the number of combinations* r *items chosen from a set of size* n*. We will refer to this as the number of combinations of* n *objects taken* r *at a time.*

Alternate Notations: $\binom{n}{r}$ or $C(n, r)$ or $C_{n,r}$.

Since every combination of r objects can be arranged in ${}_rP_r$ ways, it follows that ${}_nP_r = {}_nC_r \cdot {}_rP_r$. Hence $\dfrac{n!}{(n-r)!} = {}_nC_r \cdot r!$ and ${}_nC_r = \dfrac{n!}{r!(n-r)!}$.

Expected Value

If there is a numerical value associated with each outcome of an experiment, we may be interested in what that value will come out to be *on average*. This is called the *expected value* of the experiment. It is obtained by multiplying each numerical value by its probability and summing the results.

The numerical value associated with the outcomes is called a random variable and often denoted by X. We denote the expected value by $E(X)$.

$$E(X) = \sum x \cdot P(X = x),$$

where the sum is taken over all possible values of the random variable X.

Genetics

Definition 9 (Dominant Gene). *The gene for a given trait is called dominant if it produces the same trait whether paired with a similar or dissimilar gene.*

Definition 10 (Recessive Gene). *A gene for a given trait is called recessive if it produces its trait only when paired with a similar recessive gene.*

A *Punnett Square* is a table illustrating the results of breeding from two parents with specific genes for a given trait.