Mathematics 103
Elementary Discrete Mathematics
Monday, Wednesday 6:00-9:30

Course Overview

Interesting real-life situations involving mathematics.

- Voting Methods
- Reapportionment
- Personal Finance
- Probability
- Graphs – Paths and Networks
- Number Theory – Cryptology

Six Weeks of Classes

Eleven Classes

Two Exams (Wednesday June 9, Wednesday June 23)

Final Exam (Wednesday July 7)

Eight Other Classes

Regular Semester is Fourteen Weeks

Voting Methods

Question: How should voting be handled when one choice is to be made among several?

The Plurality Method

The candidate with the most votes wins, even if he (or she) does not receive a majority of the votes cast.

We will usually refer to voting as if it is among candidates, but the purpose of the vote is really irrelevant.

Possible Problems

• In a large field, an extremist candidate may win against the strong wishes of the majority of the electorate.

Challenge: Find an error in *Branching Out 1.1 on Pages 6-7*.

Runoff Elections

If no candidate receives a majority of the votes cast, a second plurality election is held with a designated number of the top candidates. This continues until one candidate has a majority of the votes.

The Hare Method

The candidate with the fewest votes is dropped before the runoff election.

Preference Rankings

Voters rank the candidates in order of preference.

Anomaly: If a candidate doesn't make a runoff, it's possible the candidate's supporters could have influenced a preferable outcome by voting for someone other than their first choice.

Borda's Method

Each voter ranks the candidates in order. Highest ranked candidate gets n points, next gets $n-1$ points, . . . , lowest ranked candidate gets 1 point. Total is *Borda Count*.

Arithmetic Check: If there are n candidates and v voters, the total of all the Borda Counts will be $\frac{vn(n+1)}{2}$.

Drawback: Subject to manipulation by strategic voting.

Head-to-Head Comparisons

Condorcet Winner

Definition 1 (Condorcet Winner). *A candidate who wins every head-to-head comparison is called a* Condorcet Winner*. A candidate who wins or ties every head-to-head comparison is called a* weak Condorcet Winner*.*

Drawback: There may not be a Condorcet Winner.

Single-Peaked Preference Rankings

If there is an ordering of the candidates such that the graphs of the rankings of the candidates by each voter is single-peaked then there will be a Condorcet winner.

Approval Voting

Voters indicate only approval or disapproval of each of the candidates. Each voter must both approve of at least one candidate and disapprove of at least one candidate. The winner is the candidate with the highest approval count.

Arrow's Impossibility Theorem

Definition 2 (Universal Domain). *All possible orderings of the candidates is allowed.*

Definition 3 (Pareto Optimality). *If all voters prefer candidate A to candidate B, then the group choice should not prefer candidate B to candidate A.*

Definition 4 (Non-Dictatorship). *No one individual voter's preferences totally determine the group choice.*

Definition 5 (Independence From Irrelevant Alternatives). *If a group of voters chooses candidate A to candidate B, then the addition or subtraction of other choices or candidates should not change the group choice to candidate B.*

Theorem 1 (Arrow's Impossibility Theorem). *There is no voting method based on ranking that satisfies the properties of universal domain, Pareto optimality, non-dictatorship and independence from irrelevant alternatives.*