More Final Exam Review Problems

Here are some additional review problems for the material on register machines and partial computable functions.

Note: You should be able to define/state the following items for the exam.

- $X \leq_m Y$

- A is a c.e. set

- Rice's theorem

- the s-m-n theorem (as stated in the first set of review problems)

Problem 1. Let f be a partial computable function. Explain why the function

$$g(n) = \begin{cases} 1 & \text{if } f(m) \downarrow \text{ for all } m \leq n \\ \uparrow & \text{otherwise} \end{cases}$$

is also partial computable.

Problem 2. Which sets X satisfy $X \leq_m \emptyset$?

Problem 3. Which sets X satisfy $X \leq_m \mathbb{N}$?

Problem 4. Let W be a c.e. set. Show that $W \leq_m K_0$, where $K_0 = \{\langle e, n \rangle \mid \varphi_e(n) \downarrow\}$.

Problem 5. Fix an index $e \in \mathbb{N}$ and let $A = \{i \mid \varphi_i = \varphi_e \text{ as partial functions}\}$.

- Prove that A is an index set.

- Prove that $A \neq \emptyset$ and $A \neq \mathbb{N}$.

- Prove that A is not computable.

Problem 6. Let $A = \{e \mid \text{domain}(\varphi_e) \leq_m K_0\}$.

- Prove that A is an index set.

- Explain why A is computable.

- Explain why these two facts do not contradict Rice's theorem.