Homework 2 Solutions

Problem 1

A	B	C	$A \to B$	$(A \to B) \to C$	$A \leftrightarrow B$	$\neg C$	$(A \leftrightarrow B) \vee \neg C$
T	T	T	T	T	T	F	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	T	F	T	T
F	T	T	T	T	F	F	F
F	T	F	T	F	F	T	T
F	F	T	T	T	T	F	T
F	F	F	T	F	T	T	T

DNF for $(A \to B) \to C$:

$(A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge C) \vee (\neg A \wedge \neg B \wedge C)$

DNF for $(A \leftrightarrow B) \vee \neg C$:

$(A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$
$\vee (\neg A \wedge \neg B \wedge \neg C)$

Problem 2

Same   Same

α	β	$\neg(\alpha \wedge \beta)$	$\neg\alpha \vee \neg\beta$	$\neg(\alpha \vee \beta)$	$\neg\alpha \wedge \neg\beta$
T	T	F	F	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

α	β	γ	$\alpha \to (\beta \to \gamma)$	$(\alpha \wedge \beta) \to \gamma$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Same

Problem 3 $\alpha \vee \beta$ is logically equivalent to $\neg(\neg\alpha \wedge \neg\beta)$. Therefore, since $\{\neg, \vee, \wedge\}$ is adequate, so is $\{\neg, \wedge\}$.

Problem 4 $\alpha \vee \beta$ is logically equivalent to $\neg\alpha \to \beta$. Therefore, since $\{\neg, \vee\}$ is adequate, so is $\{\neg, \to\}$.

$\boxed{\text{Problem 5}}$ $\neg\alpha$ is logically equivalent to $\alpha | \alpha$.

$\alpha \wedge \beta$ is logically equivalent to $\neg(\alpha|\beta)$, which is logically equivalent to $(\alpha|\beta)|(\alpha|\beta)$. Therefore, since $\{\neg, \wedge\}$ is adequate, $\{|\}$ is adequate as well.

$\boxed{\text{Problem 6}}$ Let A be a propositional letter. We want to show there is no proposition built from A, $\wedge$ and $\vee$ which is logically equivalent to $\neg A$. To do this, we show that for any proposition σ built from A, $\wedge$ and $\vee$, the value in the truth table for σ is T in row when A is T. More formally, we show that for any valuation $\mathcal{V}$ with $\mathcal{V}(A) = T$, we must have $\mathcal{V}(\sigma) = T$. We proceed by induction on how σ is built from A, $\wedge$ and $\vee$.

<u>Base Case</u>: σ is just A. In this case

$\mathcal{V}(\sigma) = \mathcal{V}(A) = T$, so $\mathcal{V}(\sigma) = T$ as required.

Induction Case: We consider case when σ is constructed using $\wedge$ or $\vee$. Thus, we will have cases for when σ has form $\alpha \wedge \beta$ and when σ has form $\alpha \vee \beta$.

Induction Hypothesis: If $\mathcal{V}(A) = T$, then $\mathcal{V}(\alpha) = \mathcal{V}(\beta) = T$.

Show: If $\mathcal{V}(A) = T$, then $\mathcal{V}(\sigma) = T$.

Case 1. σ has form $\alpha \vee \beta$.

Assume $\mathcal{V}(A) = T$. By induction hypothesis $\mathcal{V}(\alpha) = \mathcal{V}(\beta) = T$. By the truth table for $\alpha \vee \beta$ (which is σ),

$\mathcal{V}(\sigma) = T$.

Case 2. σ has form $\alpha \wedge \beta$.

Assume $\mathcal{V}(A) = T$. By induction hypothesis, $\mathcal{V}(\alpha) = \mathcal{V}(\beta) = T$. By truth table for $\alpha \wedge \beta$ (which is σ)

$\mathcal{V}(\sigma) = T$.

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