

Math 3265 Homework 6: Due Friday April 20

For the following exercises, you will construct register machines that compute various functions. You are welcome to use the pseudo-instructions we introduced in class (e.g. goto, copy, erase, etc.) as well as any functions that we gave register machines for (e.g. add, multiply, etc.). It would be helpful to me when I'm reading your register machines if you included a short explanation of how your register machine is computing the given function.

Problem 1. Give a register machine to compute the function

$$f(n) = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

Problem 2. Give a register machine which will compute the function

$$g(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

Hint. If you find it helpful, you can use the function f from Problem 1 to help you.

Problem 3. Give a register machine that computes the following function

$$h(n, m) = \begin{cases} 1 & \text{if } n < m \\ 0 & \text{if } n \geq m \end{cases}$$

Problem 4. Give a register machine for the following function

$$f(n, m) = \begin{cases} 0 & \text{if } n = m = 0 \\ 1 & \text{otherwise} \end{cases}$$

For the next three problems, recall that a set $A \subseteq \mathbb{N}$ is *computable* if the characteristic function χ_A of A is computable. That is, there is a register machine taking a single input $M(x)$ such that for all n , $M(n)$ halts and

$$M(n) = \begin{cases} 1 & \text{if } n \in A \\ 0 & \text{if } n \notin A \end{cases}$$

Problem 5. Let A be a computable set. Explain why $\mathbb{N} \setminus A$ is also computable.

Problem 6. Let A and B be computable sets. Explain why $A \cup B$ and $A \cap B$ are computable.

Hint for Problems 5 and 6. You are welcome to answer these questions by describing how to use register machines to compute characteristic functions for the desired sets. Alternately, there are very short proofs for both problems if you remember that computable functions are closed under composition. For example, the function from Problem 4 might help you in Problem 6.