Math 3265 Homework 2: Due Friday February 9

Problem 1. Give the truth table and disjunctive normal form for $(A \to B) \to C$ and for $(A \leftrightarrow B) \vee (\neg C)$.

Problem 2. Let α, β and γ be propositions.

2(a). Show that $\neg(\alpha \wedge \beta)$ is logically equivalent to $\neg\alpha \vee \neg\beta$.

2(b). Show that $\neg(\alpha \vee \beta)$ is logically equivalent to $\neg\alpha \wedge \neg\beta$.

2(c). Show that $\alpha \to (\beta \to \gamma)$ is logically equivalent to $(\alpha \wedge \beta) \to \gamma$.

Problem 3. Prove that $\{\neg, \wedge\}$ is an adequate set of connectives.

Problem 4. Prove that $\{\neg, \to\}$ is an adequate set of connectives.

Problem 5. The Sheffer stroke is the connective corresponding to "not both $\cdots$ and $\cdots$". It is given formally by the following truth table.

α	β	$\alpha\|\beta$
T	T	F
T	F	T
F	T	T
F	F	T

Prove that $\{|\}$ is an adequate set of connectives.

Problem 6. Prove that $\{\wedge, \vee\}$ is not an adequate set of connectives.

Practice Problems

You do not need to hand in the following problems. They are extra practice problems similar to the ones on this homework to help you prepare for the exams.

The point of the first problem is to show that we can write expressions such as $\alpha_0 \wedge \alpha_1 \wedge \cdots \wedge \alpha_n$ and $\alpha_0 \vee \alpha_1 \vee \cdots \vee \alpha_n$ without worrying about how the parentheses are arranged or what order we list the terms in.

Practice Problem 1. Let α, β and γ be propositions.

1(a). Show that $((\alpha \wedge \beta) \wedge \gamma)$ is logically equivalent to $(\alpha \wedge (\beta \wedge \gamma))$; and that $\alpha \wedge \beta$ is logically equivalent to $\beta \wedge \alpha$.

1(b). Show that $((\alpha \vee \beta) \vee \gamma)$ is logically equivalent to $(\alpha \vee (\beta \vee \gamma))$; and that $\alpha \vee \beta$ is logically equivalent to $\beta \vee \alpha$.

Practice Problem 2. The connective corresponding to "neither $\cdots$ nor $\cdots$" is often denoted $\downarrow$. It is given formally by the following truth table.

α	β	$\alpha \downarrow \beta$
T	T	F
T	F	F
F	T	F
F	F	T

Prove that $\{\downarrow\}$ is an adequate set of connectives.

Practice Problem 3. Prove that $\{\vee, \rightarrow, \leftrightarrow\}$ is not an adequate set of connectives.