Math 2142 Homework 9: Due Friday April 13

Problem 1. In Exercises 12.4 of the textbook, do Exercises 1, 2, 5, 6 and 7.

Hint for Exercise 5. You are given $A = \langle 2, 1\rangle$ and $B = \langle 1, 3\rangle$, so for any $x, y \in \mathbb{R}$,

$$xA + yB = x\langle 2, 1\rangle + y\langle 1, 3\rangle = \langle 2x + y, x + 3y\rangle$$

Given an arbitrary vector $C = \langle c_1, c_2\rangle$, you need to find x and y such that $C = xA + yB$. That is, you need to find x and y so that

$$\langle c_1, c_2\rangle = \langle 2x + y, x + 3y\rangle$$

Writing this out as components, you get the following system of equations

$$\begin{aligned} c_1 &= 2x + y \\ c_2 &= x + 3y \end{aligned}$$

Solve this system of equations by expressing x and y in terms of c_1 and c_2.

Hint for Exercise 6(b). For 6(b), you need to show that the only solution to

$$\langle 0, 0, 0\rangle = x\langle 1, 1, 1\rangle + y\langle 0, 1, 1\rangle + z\langle 1, 1, 0\rangle$$

is $x = y = z = 0$. To find find x, y and z solving this equation, combine the vectors to get

$$\langle 0, 0, 0\rangle = \langle x + z, x + y + z, x + y\rangle$$

and separate into components

$$\begin{aligned} 0 &= x + z \\ 0 &= x + y + z \\ 0 &= x + y \end{aligned}$$

Solve this system of equations to see that the only solution is $x = y = z = 0$.

Hint for Exercise 7(b). Follow the same pattern as for 6(b). When you solve the system of equations, you should fine that there are many solutions, not just $x = y = z = 0$.

Hint for Exercise 7(c). Follow the pattern of 6(b) and 7(b) except with $\langle 1, 2, 3\rangle$ instead of $\langle 0, 0, 0\rangle$. This time, you should find that there is no solution!

Problem 2. Let $\overline{u}$, $\overline{v}$ and $\overline{w}$ be vectors in V_n and let $c \in \mathbb{R}$ be a scalar. Prove that

$$\begin{gathered} c(\overline{u} + \overline{v}) = c\overline{u} + c\overline{v} \\ \overline{u} \cdot (\overline{v} + \overline{w}) = \overline{u} \cdot \overline{v} + \overline{u} \cdot \overline{w} \\ \|c\overline{v}\| = |c|\, \|\overline{v}\| \end{gathered}$$

by looking at the components of each vector.

Problem 3. In Exercises 12.8 of the textbook, do Exercises 1, 5, 6, 9, 19 and 20.

Hint for Exercise 5. Let $C = \langle x, y, z \rangle$. You want C to satisfy $A \cdot C = 0$ and $B \cdot C = 0$. You have

$$A \cdot C = \langle 2, 1, -1 \rangle \cdot \langle x, y, z \rangle = 2x + y - z$$

so you know you need $2x + y - z = 0$. Write out $B \cdot C = 0$ and you will get another equation with x, y and z. Solve this system of equations to find values of x, y and z giving a vector C with the desired property.

Problem 4. In Exercises 12.11 of the textbook, do Exercises 1, 2 and 5.

Hint for Exercise 5. You are given the three vertices of the triangle in $\mathbb{R}^3$. Use these points to find the vectors representing the sides of the triangle. Once you have the vectors for the sides of the triangle, you can find the angles between them.