Math 2142 Homework 5 Part 2: Due Friday February 23

Problem 1. Consider a pair of linear 2nd order differential equations with the same left side.

$$\begin{aligned} y'' + P_1(x)\, y' + P_2(x)\, y &= Q_1(x) \\ y'' + P_1(x)\, y' + P_2(x)\, y &= Q_2(x) \end{aligned}$$

Prove that if $y_1(x)$ is a solution to the top equation and $y_2(x)$ is a solution to the bottom equation, then $y_1(x) + y_2(x)$ is a solution to

$$y'' + P_1(x)\, y' + P_2(x)\, y = Q_1(x) + Q_2(x)$$

Hint. I would let $L(y) = y'' + P_1(x)y' + P_2(x)y$ and use the fact that $L(y)$ is a linear operator to write a very short proof for this problem.

Problem 2(a). Find the general solution for

$$y'' + y = e^x + x^3 + x + 2$$

Hint. By Problem 1, you can separate this problem into three steps. First, find the general solution to the homogeneous equation. Second, find a particular solution to $y'' + y = e^x$. Third, find a particular solution to $y'' + y = x^3 + x + 2$. The sum of these three parts will be the general solution.

2(b). Find the particular solution to the equation in 2(a) satisfying $y(0) = 2$ and $y'(0) = 0$.

Problem 3. Use Laplace transforms to solve the following initial value problems.

$$y'' - y' - 6y = 0 \text{ with } y(0) = 1 \text{ and } y'(0) = -1$$
$$y'' - 2y' + 2y = 0 \text{ with } y(0) = 0 \text{ and } y'(0) = 1$$
$$y'' + 9y = 1 \text{ with } y(0) = y'(0) = 0$$
$$y'' + 4y = \sin 3t \text{ with } y(0) = y'(0) = 0$$