

**Math 2142 Homework 4 Part 1, Due Friday February 16**

**Problem 1.** Solve the following separable initial value problems.

$$\begin{aligned}\frac{dy}{dx} &= (1 - 2x)y^2 && \text{with } y(0) = -1/6 \\ \frac{dy}{dx} &= xy^3(1 + x^2)^{-1/2} && \text{with } y(0) = 1\end{aligned}$$

**Problem 2.** Find the general solutions for the following separable differential equations. For the second one, you do not need to solve explicitly for  $y$ .

$$\begin{aligned}\frac{dy}{dx} &= \frac{x^2}{y(1 + x^3)} && \text{(you can assume that } x > 0) \\ \frac{dy}{dx} &= \frac{2x}{ye^y - x^2ye^y} && \text{(you can assume that } -1 < x < 1)\end{aligned}$$

For the next problem, you will need to solve equations of the form  $y^2 + by + c = f(x)$  explicitly for  $y$ . Since the left side of this equation is a quadratic function of  $y$ , you can solve for  $y$  by completing the square on the left side to get something of the form  $(y + b/2)^2 + d = f(x)$ . From here, you can directly solve for  $y$ .

**Problem 3(a).** Solve the following algebraic equation explicitly for  $y$ .

$$y^2 - 2y + 4 = x^4 + x^2 + 6$$

**3(b).** Solve the following separable initial value problem and give the solution explicitly as a function  $y(x)$ .

$$\frac{dy}{dx} = \frac{3x^2 + e^x}{2y - 4} \quad \text{with } y(0) = 1$$

**Problem 4.** Find the general solution for the following linear differential equations. If an initial condition is given, find the specific solution as well.

$$\begin{aligned}\frac{dy}{dx} + 2xy &= 4e^{-x^2} && \text{with } y(0) = 3 \\ \frac{dy}{dt} &= \frac{y}{t} + 2t^2 && \text{for } t > 0 \\ \frac{dy}{dx} + y &= 3x \\ \frac{dy}{dx} + 3y &= \sin 2x\end{aligned}$$

**Practice Problems.** You do not need to hand in these problems, but they are good practice.

Exercises 8.5: 1, 2, 4, 5.

Exercises 8.24: 1-9