Math 2142 Homework 1: Due Friday January 26

Problem 1. Prove that for any $n \geq 2$, the log function $\ln(x)$ grows slower than $\sqrt[n]{x}$ by proving that

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[n]{x}} = 0$$

(You do not need induction. You can calculate this limit directly using L'Hopital's Rule.)

Problem 2. Find the following limits.

2(a).

$$\lim_{x\to\infty} \frac{3x^2 - x + 7}{e^x}$$

2(b).

$$\lim_{x\to\infty} \frac{(\ln(x))^2}{\sqrt[3]{x}}$$

2(c).

$$\lim_{x\to\infty} \frac{\sin(x) - x}{x^3}$$

2(d).

$$\lim_{x\to 0+} \sqrt{x}\ln(x)$$

2(e).

$$\lim_{x\to 0^+} \sin(x)\ln(x)$$

2(f).

$$\lim_{x\to\infty} xe^{1/x} - x$$

2(g).

$$\lim_{x\to 0^+} x^{\sqrt{x}}$$

2(h).

$$\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x$$

Problem 3. Calculate the following convergent integrals.

$$\int_1^\infty e^{-2x}\,dx$$

$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx$$

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx$$

Problem 4. Use the Comparison Theorem to determine if the following integrals converge or diverge. You do *not* need to calculate the exact value of the integrals.

$$\int_e^\infty \frac{\ln(x)}{x}\,dx$$

$$\int_1^\infty \frac{1}{x+e^{2x}}\,dx$$

$$\int_1^\infty \frac{x}{\sqrt{1+x^6}}\,dx$$

$$\int_2^\infty \frac{x+2}{\sqrt{x^3-1}}\,dx$$

$$\int_1^\infty \frac{\sin^2 x}{1+x^2}\,dx$$

Practice Problems. Here is a list of additional problems you can work on for more practice. You do not need to hand these in.

- Exercises 7.13: 1,2,4,5,7.
- Exercises 10.24: 1, 2, 3, 4.

Practice Problem 1. Let $p > 0$. Show that $\int_0^1 1/x^p\,dx$ converges if and only if $0 < p < 1$ using the following three steps.

(a). Show that if $0 < p < 1$, then $\int_0^1 1/x^p\,dx$ converges.

(b). Show that $\int_0^1 1/x\,dx$ diverges.

(c). Show that if $p > 1$, then $\int_0^1 1/x^p\,dx$ diverges.

Practice Problem 2. Use the Comparison Theorem and Practice Problem 1 to determine if the following integral converges or diverges. You do *not* need to calculate the exact value of the integral.

$$\int_0^1 x^{-1/2} e^x\,dx$$