Terms to know for Math 2142 final exam

For the final exam, you should be able to state precisely the following definitions and results. In these definitions, let $S = \{\overline{v}_1, \ldots, \overline{v}_k\}$ be a set of vectors in $\mathbb{R}^n$.

- $\overline{u} \in \mathrm{Span}(S) \Leftrightarrow$ there are scalars $c_1, \ldots, c_k$ such that $\overline{u} = c_1\overline{v}_1 + \cdots + c_k\overline{v}_k$.
- S is linearly independent $\Leftrightarrow$ if $c_1\overline{v}_1 + \cdots + c_k\overline{v}_k = \overline{0}$, then $c_1 = \cdots = c_k = 0$.
- S is a basis for $\mathbb{R}^n \Leftrightarrow S$ is linearly independent and $\mathrm{Span}(S) = \mathbb{R}^n$.
  - On the exam, you are welcome to use the fact that S is a basis for $\mathbb{R}^n$ if and only if S is linearly independent and $|S| = n$. But the definition above is the official definition of a basis.
- Triangle Inequality: For vectors $\overline{u}, \overline{v} \in \mathbb{R}^n$, $\|\overline{u} + \overline{v}\| \leq \|\overline{u}\| + \|\overline{v}\|$.
- Cauchy-Schwarz Inequality: For vectors $\overline{u}, \overline{v} \in \mathbb{R}^n$, $|\overline{u} \cdot \overline{v}| \leq \|\overline{u}\| \, \|\overline{v}\|$.