Math 2142 Exam 2 Review Problems

Problem 1. Find limits of the following sequences.

$$a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$$
$$a_n = ne^{-n}$$
$$a_n = \cos n\pi$$
$$a_n = \frac{(-1)^n n}{n^2+1}$$
$$a_n = \frac{4^n}{n!}$$
$$a_n = \sqrt[n]{n}$$
$$a_n = \sum_{k=0}^{n} \frac{1}{2^k}$$

Problem 2. Let a_n be a sequence such that $\lim_{n\to\infty} a_n = L > 0$. Prove that there is an N such that for all $n \geq N$, $a_n > 0$.

Problem 3. Find the exact value of the following convergent series.

$$\sum_{k=0}^{\infty} \frac{(-1)^n 5^n}{6^n}$$
$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$
$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$
$$\sum_{n=0}^{\infty} (\sin 1)^n$$

Problem 4. Let $\sum a_n$ be an absolutely convergent series. Prove that

$$\Big|\sum_{n=1}^{\infty} a_n\Big| \leq \sum_{n=1}^{\infty} |a_n|$$

Hint. Use the triangle inequality.

Problem 5. For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

Problem 6. For which $r > 0$ is the following series convergent?

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^r}$$

Problem 7. Find power series representations for the following functions and give their interval of convergence.

$$\frac{3}{1-x^4}$$
$$\frac{x}{4x+1}$$
$$\frac{x}{4+x}$$
$$\frac{1}{9+x^2}$$
$$\frac{x}{4-x}$$

Hint. For the next to last one, factor 9 out of the denominator to put it into a more helpful form. Use a similar trick for the last one.

Problem 8. Show that the functions

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \text{ and } g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

are solutions to the differential equation $y'' + y = 0$.

Problem 9. Find the interval of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$$
$$\sum_{n=1}^{\infty} (-1)^n 4^n x^n$$
$$\sum_{n=2}^{\infty} \frac{x^n}{4^n \ln n}$$
$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$$

Problem 10. Approximate the following integrals.

$$\int_0^1 x \cos x^3 \, dx \text{ within } 10^{-4}$$

$$\int_0^{0.2} \sin x^2 \, dx \text{ within } 10^{-3}$$

$$\int_0^{0.5} x^2 e^{-x} \, dx \text{ within } 10^{-5}$$