

The comparison series for the Limit Comparison Test is $\sum b_n$, where

$$b_n = \frac{\sqrt{n^3}}{3n^3} = \frac{n^{3/2}}{3n^3} = \frac{1}{3n^{3/2}}$$

EXAMPLE 3 $\sum_{n=1}^{\infty} ne^{-n^2}$

Since the integral $\int_1^{\infty} xe^{-x^2} dx$ is easily evaluated, we use the Integral Test. The Ratio Test also works.

EXAMPLE 4 $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4 + 1}$

Since the series is alternating, we use the Alternating Series Test.

EXAMPLE 5 $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

Since the series involves $k!$, we use the Ratio Test.

EXAMPLE 6 $\sum_{n=1}^{\infty} \frac{1}{2 + 3^n}$

Since the series is closely related to the geometric series $\sum 1/3^n$, we use the Comparison Test.

12.7 Exercises

1–38 ■ Test the series for convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$

2. $\sum_{n=1}^{\infty} \frac{n-1}{n^2 + n}$

19. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

20. $\sum_{k=1}^{\infty} \frac{k+5}{5^k}$

3. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

4. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2 + n}$

21. $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

22. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$

5. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

6. $\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n} \right)^n$

23. $\sum_{n=1}^{\infty} \tan(1/n)$

24. $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$

7. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

8. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$

25. $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

26. $\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$

9. $\sum_{k=1}^{\infty} k^2 e^{-k}$

10. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

27. $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$

28. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

11. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$

12. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 25}$

29. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$

30. $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$

13. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

14. $\sum_{n=1}^{\infty} \sin n$

31. $\sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$

32. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$

15. $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$

16. $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$

33. $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

34. $\sum_{n=1}^{\infty} \frac{1}{n + n \cos^2 n}$

17. $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$

18. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n-1}}$

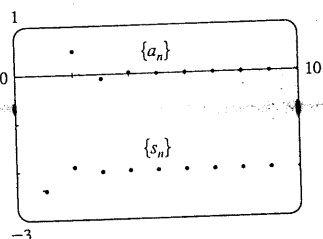
35. $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$

36. $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

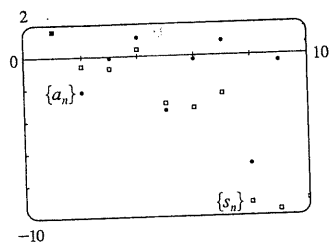
37. $\sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)^n$

38. $\sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)$

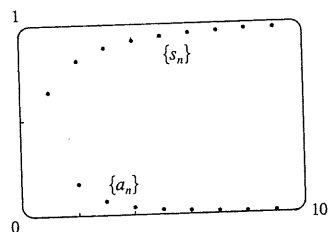
3. -2.40000, -1.92000
 -2.01600, -1.99680,
 -2.00064, -1.99987
 -2.00003, -1.99999
 -2.00000, -2.00000
 Convergent, sum = -2



5. 1.55741, -0.62763,
 -0.77018, 0.38764
 -2.99287, -3.28388
 -2.41243, -9.21214
 -9.66446, -9.01610
 Divergent

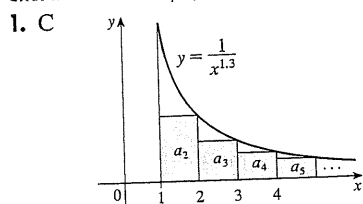


7. 0.64645, 0.80755,
 0.87500, 0.91056,
 0.93196, 0.94601,
 0.95581, 0.96296,
 0.96838, 0.97259
 Convergent, sum = 1



9. (a) C (b) D
 11. 9 13. D 15. 15 17. $\frac{1}{7}$ 19. D 21. D
 23. $\frac{3}{2}$ 25. D 27. $\frac{3}{2}$ 29. D 31. D 33. $\frac{7}{2}$
 35. $\frac{2}{9}$ 37. $\frac{1138}{333}$ 39. 41,111/333,000
 41. $-3 < x < 3; x/(3-x)$ 43. $-\frac{1}{4} < x < \frac{1}{4}; 1/(1-4x)$
 45. all $x; \frac{2}{2-\cos x}$ 47. $\frac{1}{4}$
 49. $a_1 = 0, a_n = 2/[n(n+1)]$ for $n > 1$, sum = 1
 51. (a) $S_n = D(1-c^n)/(1-c)$ (b) 5
 53. $(\sqrt{3}-1)/2$ 55. $1/[n(n+1)]$
 57. The series is divergent.
 63. $\{s_n\}$ is bounded and increasing.
 65. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$
 67. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}; [(n+1)! - 1]/(n+1)!$ (c) 1

Exercises 12.3 page 765



3. C 5. D 7. C 9. D 11. C 13. C 15. C

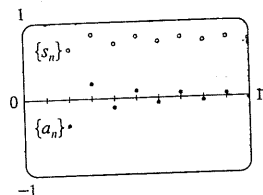
17. D 19. C 21. D 23. C 25. $p > 1$
 27. $p < -1$ 29. $(1, \infty)$
 31. (a) 1.54977, error ≤ 0.1 (b) 1.64522, error ≤ 0.005
 (c) $n > 1000$
 33. 2.61 39. $b < 1/e$

Exercises 12.4 page 770

1. (a) Nothing (b) C
 3. C 5. C 7. D
 9. C 11. D 13. C 15. C 17. D 19. C
 21. D 23. C 25. D 27. C 29. C 31. D
 33. 0.567975, error ≤ 0.0003 35. 0.76352, error < 0.001
 45. Yes

Exercises 12.5 page 775

1. (a) A series whose terms are alternately positive and negative
 (b) $0 < b_{n+1} \leq b_n$ and $\lim_{n \rightarrow \infty} b_n = 0$, where $b_n = |a_n|$
 (c) $|R_n| \leq b_{n+1}$
 3. C 5. C 7. D 9. C 11. C 13. D
 15. C 17. C 19. D
 21. 1.0000, 0.6464,
 0.8389, 0.7139, 0.8033,
 0.7353, 0.7893, 0.7451, 0.7821,
 0.7505; error < 0.0275



23. 10 25. 7 27. 0.9721 29. 0.0676
 31. An underestimate 33. p is not a negative integer
 35. $\{b_n\}$ is not decreasing

Exercises 12.6 page 781

Abbreviations: AC, absolutely convergent;
 CC, conditionally convergent

1. (a) D (b) C (c) May converge or diverge
 3. AC 5. CC 7. D 9. AC 11. AC 13. AC
 15. AC 17. CC 19. AC 21. D 23. AC
 25. AC 27. D 29. D 31. (a) and (d)
 35. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521 (b) $n \geq 11$, 0.693109

Exercises 12.7 page 784

1. D 3. C 5. C 7. D 9. C 11. C 13. C
 15. C 17. D 19. C 21. C 23. D 25. C
 27. C 29. C 31. D 33. C 35. C 37. C

Exercises 12.8 page 789

1. A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$, where x is a variable and a and the c_n 's are constants
 3. 1, $[-1, 1]$ 5. 1, $[-1, 1]$ 7. $\infty, (-\infty, \infty)$ 9. $\frac{1}{4}, (-\frac{1}{4}, \frac{1}{4})$
 11. $\frac{1}{2}, (-\frac{1}{2}, \frac{1}{2})$ 13. 4, $(-4, 4)$ 15. 1, $(0, 2)$
 17. 2, $(-4, 0]$ 19. $\infty, (-\infty, \infty)$ 21. $b, (a-b, a+b)$