Problem 1. Explain why \( \{ x \in \mathbb{Z} \mid 0 \leq x \text{ and } x \leq 2 \} = \{ x \in \mathbb{R} \mid x^3 - 3x^2 + 2x = 0 \} \).

*Hint.* You can figure out exactly which elements are in each set.

Problem 2. Explain why \( \{ x \in \mathbb{R} \mid x^3 - 2x^2 + x = 2 \} \neq \{ 0, 1, 2 \} \) and find the elements in \( \{ x \in \mathbb{R} \mid x^3 - 2x^2 + x = 2 \} \cap \{ 0, 1, 2 \} \).

*Hint.* You do not need to figure out exactly which elements are in \( \{ x \in \mathbb{R} \mid x^3 - 2x^2 + x = 2 \} \) to do either part of this problem.

Problem 3. Prove that the following properties hold for all sets \( A, B \) and \( C \).

3(a). If \( A \subseteq B \) and \( A \subseteq C \), then \( A \subseteq B \cap C \).

3(b). If \( A \subseteq B \cup C \) and \( A \cap B = \emptyset \), then \( A \subseteq C \).

Problem 4. Let \( A \) and \( B \) be sets such that \( A \cap B = \emptyset \). What is \( A - B \)? Explain your answer.

Problem 5. Prove the following theorems.

- **Theorem I.7. Cancellation law for multiplication.** Let \( a, b, \) and \( c \) be real numbers. If \( ab = ac \) and \( a \neq 0 \), then \( b = c \).

- **Theorem I.22.** Let \( a, b, \) and \( c \) be real numbers. If \( a < b \) and \( c < 0 \), then \( ac > bc \).

*Hint.* Theorem I.7 is similar to Theorem I.1 which was proved in class (also see page 19 of the textbook). Theorem I.22 is similar to Theorem I.19, the proof of which is in the textbook, see page 21. Also, for the proof of Theorem I.22 it is assumed that all the algebraic properties work and, basically, one should only use the order axioms to prove the statement, that is, Axioms 7, 8, and 9.

Problem 6. How many real numbers having the property of 0 in Axiom 4 are there? Explain your answer.

Problem 7. How many real numbers having the property of 1 in Axiom 4 are there? Explain your answer.

*Hint.* To solve Problem 6, you need to read the proof of Theorem I.1 on page 19 carefully. Then adapt the reasoning to the case of Problem 7.
The Guide for Quiz 1

Quiz 1 will take place on Friday, September 6th and it will be during last 25-30 minutes of the class (that is, if you are done with the quiz, you are free to go). Quiz 1 will have 2 questions. The first one is to prove that $\sqrt{2}$ is irrational (to prove that you need to solve 10 and 11 from I 3.12 Exercises, page 28). The second part will be concerned with the set theory. The practice problems for that part are 1, 2, 3, 5, 7, 11, 12, 14, 15 in I.2.5 Exercises (pages 15-16).