HW 1

1.1.3

a) 2, linear inhomog

b) 2, linear homog

c) 3 nonlinear

d) 2 linear inhomog

e) 2 linear homog

f) 1 ~~linear~~ nonlinear

g) 1 linear homog

h) 4 non linear

b/c $= 1$.

1.1.7

$1+x,\ 1-x,\ 1+x+x^2$ lin indep

b/c $\quad a(1+x) + b(1-x) + c(1+x+x^2) = 0$

$\Rightarrow$ (sub $x=0$) $\quad a+b+c=0$

and (diff then sub $x=0$) $\quad a-b+c=0$

and (diff twice) $\quad 2c=0$

So $c=0$ and $\left.\begin{array}{l} a+b=0 \\ a-b=0 \end{array}\right\} \Rightarrow a=b=0$

1.2.3

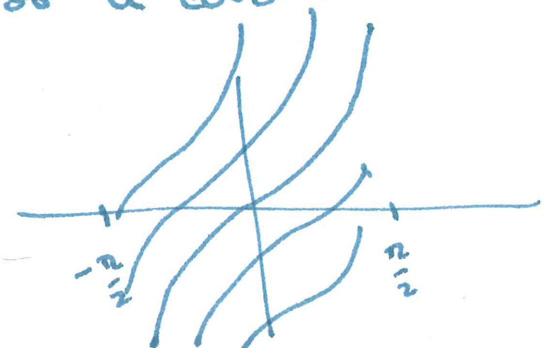
$(1+x^2)u_x + u_y = 0$

View as $(1+x^2, 1)\cdot(u_x, u_y) = 0$

So u const on curves with $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

$\Rightarrow y = \tan^{-1}x + c$

$\Rightarrow u = f(c) = f(y - \tan^{-1}x)$



1.2.6 $\quad \sqrt{1-x^2}\, u_x + u_y = 0 \qquad u(0,y) = \cancel{e^{y^2}}\; y$

$$= \left(\sqrt{1-x^2},\, 1\right) \cdot \left(u_x, u_y\right)$$

So u is const on curves with $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

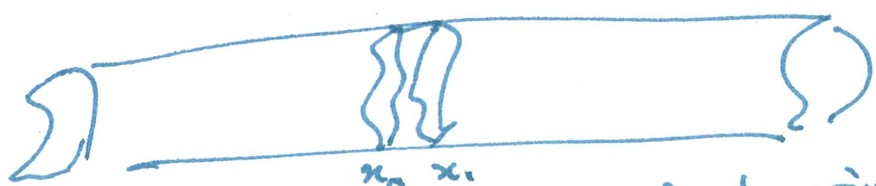
$$\Rightarrow \quad y = \sin^{-1} x + c$$

Hence $u = f(y - \sin^{-1} x)$

and then $u(0,y) = f(y) = \cancel{e^{y^2}}\; y$

$$\Rightarrow \quad u(x,y) = \cancel{e^{(y-\sin^{-1}x)^2}}\; y - \sin^{-1} x .$$

1.3.3



Tube cross section const, with area A & perim P.

Have $\iiint_R u(x,t)\, dx\,dy\,dz = \int_{x_0}^{x_1} A\, u(x,t)\, dx$

for R region x_0 to x_1

and so amt of heat in R is $Ac\rho \int_{x_0}^{x_1} u(x,t)\, dx$

and rate of change of this wrt t is $Ac\rho \int_{x_0}^{x_1} u_t\, dx$

But this must equal bdy flux, which is in 3 pieces

$\kappa \iint_{\text{cross section}} u_x(x_1,t) = A u_x(x_1,t)$ $\quad$ with κ is heat conductivity of tube

$\kappa \iint_{\text{cross sect}} -u_x(x_0,t) = -A u_x(x_0,t)$

$\mu \iint_{\text{surface of tube btwn } x_0 \,\&\, x_1} (u-T)\, dx = \mu P \int_{x_0}^{x_1} (u-T)\, dx$ $\quad$ b/c perimeter length is const & u doesnt vary on perim.

μ is conductiv. of bdy

So $\displaystyle\int_{x_0}^{x_1} Ac\rho u_t(x,t) - Au_{xx} - \mu P(u-T)\,dx = 0$
for every x_0, x_1

$$\Rightarrow \quad u_t = \frac{1}{c\rho} u_{xx} + \frac{\mu P}{Ac\rho}(u-T).$$

<u>1.3.6</u> This is a strange problem b/c the coord along axis of cylinder is irrelevant. We have

$$u(x,y,t) \quad \text{with} \quad u_t = k\Delta u.$$

and assume u depends on x,y only via $r = \sqrt{x^2+y^2}$.

Then
$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} = u_r \frac{x}{r}$$

$$u_{xx} = \frac{u_r}{r} + \left(\frac{u_{rr}}{r} - \frac{u_r}{r^2}\right)\frac{x^2}{r}$$

and similarly for u_{yy}, so

$$\Delta u = \frac{2u_r}{r} + \left(\frac{u_{rr}}{r} - \frac{u_r}{r^2}\right)\left(\frac{x^2+y^2}{r}\right)$$

$$= u_{rr} + \frac{u_r}{r}.$$

Hence $\quad u_t = k\left(u_{rr} + \dfrac{u_r}{r}\right).$