MATH 2710W

Purpose:

This course exists to prepare you for proof-based courses in mathematics. Some of the mathematical content may be new to many of you, but it is not the focus of the course. The the main thing you are intended to learn is how to read, write, and understand mathematical proofs.

The focus on proof makes this course very different to many of the previous math courses you have taken, in that memorizing methods and identifying which method goes with which problem is not a very helpful strategy. Instead, problems will often feel more like exercises in translation from english to math, or in mathematical grammar: you will spend a lot of time figuring out how to arrange symbols and words to make unambiguous and correct mathematical arguments.

Text:

The text is "An introduction to Mathematical Thinking" by Gilbert and Vanstone. Readings and homework are assigned from the text, so you need access to it.

Website:

A copy of this document, as well as other potentially useful information, including links for installing and using $\mathbb{A}T_{\mathbb{E}}X$, may be found on the course website:

www.math.uconn.edu/~rogers/math2710f18

Note that on this website there are documents for both the W and non-W versions of the course – make sure you download the right ones!

Expectations:

It is expected that you will attend all classes, and midterms, and complete all homework. Many questions on midterms will be similar or identical to homework questions, so doing the homework will also be beneficial to you on exams. You are expected to read the book outside class. Reading to be completed before each class is listed in the Syllabus and Schedule.

Anything covered in class may appear on an exam, whether or not it is in the book. Anything in the sections of the book that are listed on the schedule may appear on an exam, whether or not it was discussed in class. It cannot be emphasized too much that mathematics is learned by doing, so the work you do outside class will be critical to your success in this course.

Grading:

Since this course plays a critical role in preparing you for proof-based material in later courses, I am required (by department policy) to give you a grade below C if, in my assessment, you are not ready to take those later courses. You need a C or better in this course to move on to those courses. This rule was put in place to protect students: before this rule was implemented, most students who passed this course with a low grade went on to fail one or more proof-based courses, making it hard for them to complete a math major.

Your grade will be based on homework (10%), as well as two midterms and your portfolio (30% each). There is no final exam, but you must receive a passing grade on the writing portfolio in order to pass the course (see below for details).

One or two questions from each weekly homework will be graded, and there will be time in class to discuss problems that you feel you did not understand. You are also strongly urged to attend office hours to discuss questions from the homework. All exams will include at least one question requiring you to replicate a proof from the book; a list of the proofs that might occur will be provided. Moreover, at least 50% of the questions on all exams will be minor modifications of homework exercises. It follows that if

you completely understand the proofs in the book and all the homework exercises then you can certainly pass the class, provided you pass the writing portfolio component.

Homework and Latex:

Homework will be assigned weekly, according to the Syllabus and Schedule. Reading and homework assignments may be modified during the semester if we progress more rapidly or more slowly through the text than anticipated.

Homework must be typed. You can use any system that produces decent output, but the easiest way to type math well is to use Latex. Latex is a markup language (like html). You will need a Latex compiler and editor, and will probably find it helpful to read some introductory material about how to use Latex. Some good sources are listed on the website.

Getting Help:

The main ways to get help are to ask questions in class and to come to my office hours. If you have any doubt about your work you should get help early and often. Do not leave this until the first midterm!

Writing Portfolio: This is a W course. W courses exist to teach writing of a nature specific to the department offering the course. In this case the writing is of a technical type applicable to mathematics. During the semester you will produce a portfolio of proofs that must be submitted at least once as a draft for comments and corrections, and may be submitted for comments up to three times. The completed portfolio, including drafts and final versions of each question, will be submitted in lieu of a final exam. You cannot receive a passing grade for the course without achieving a passing grade on the portfolio. More details about what will go into your portfolio are in a separate document on the class website.

Schedule:

The following is a draft of our schedule. It is incomplete, and you will get updated versions from time to time.

- Week 1 (8/28, 8/30) Logic, Sets, and Quantifiers. Reading: Sections 1.1–1.4.
 Exercises to hand in: pages 20–23: 2,3,5,6,8,12,17–21,29–32 on Thursday 9/6. Portfolio problem: page 23 question 81 on Thursday 9/6.
- Week 2 (9/4, 9/6) Proof techniques, counterexamples. Reading: Sections 1.5, 1.6
 Exercises to hand in: page 20-23: 35,36,38,49,50,53,54 on Thursday 9/13.
 Portfolio problem: page 22 question 75 on Thursday 9/13. Also give an example of a function f and values a and L for which the negated statement holds (you could think of this as a counterexample to your function f having limit L as x → a).
- Week 3 (9/11, 9/13) Integers, Divisibility, Diophantine equations. Reading: Sections 2.1–2.3 Exercises to hand in: page 50–53: 2,6,9,10,11,15,16,20,22,27,28,38,40,44,51 on Thursday 9/20. Portfolio problems: page 52-53, questions 74 and 83 on Thursday 9/20.
- Week 4 (9/18, 9/20) Prime Numbers, Congruence, Tests for divisibility. Reading: Sections 2.5, 3.1, 3.2 Exercises to hand in: page 50–53: 93, 94,101 and pages 82–84: 2,21 on Thursday 9/27. Portfolio problems: page 50–53 questions 99, 107 on Thursday 9/27.
- Week 5 (9/25, 9/27) Equivalence relations, Modular arithmetic Reading: Sections 3.3, 3.4 Exercises to hand in: page 82–84: 24,26,30,31,42 on Thursday 10/4 (Note: small homework because of midterm).

Portfolio problem: None.

• Week 6 (10/2, 10/4)

MIDTERM ON 10/2. Covers material from chapters 1 and 2 only. It will consist of 5 or 6 questions, at least 3 of which are either taken from the homework or are small modifications of homework questions, and at least one of which will be to state and prove a theorem from the following list: Contrapositive Law 1.53, Prop 2.21, Prop 2.28, Prop 2.51, Thm 2.52, Thm 2.53. Reading: Section 4.1 (for 10/4)

Exercises to hand in: page 85-88: 68,69,80 and page 104: 9,12-18 on Thursday 10/11. Portfolio problem: page 85-88, question 79 on Thursday 10/11.

• Week 7 (10/9, 10/11)

Reading: Sections 4.2, 4.3 Exercises to hand in: page 104–109: 20.23.25.30.32.37.40.41.43.45.51.56.57.65 on Thursday 10/18. Portfollio problem: page 104–109, question 74

• Week 8 (10/16, 10/18)

Reading: Sections 5.1, 5.2, 5.4. Exercises to hand in: page 121-123 questions 4,5,6,10,24,26,30,32,33,40 on Thursday 10/25. Portfolio problems page 121-123, questions 41,42 on Thursday 10/25.

• Week 9 (10/23, 10/25)

Reading: Sections 6.1–6.4 Exercises to hand in: pages 153-161, questions 2,4,6,8,12,16,17,89,90,91 on Thursday 11/1. Portfolio Problem: None.

• Week 10 (10/30, 11/1)

Reading: Sections 6.5, 6.6 Exercises to hand in: page 153–161 questions 19,21,22,25,28,33,36,37,38,43,44,45 on Thursday 11/8.Portfolio Problem: page 159, question 99.

• Week 11 (11/6, 11/8)

Reading: From the Analysis Notes (can be downloaded from website), First section (Sequences and Limits of Sequences.

Exercises to hand in: 12.4, 12.6, 12.9. Note that all arguments should use the definition of limit, not any theorems about the algebraic properties of limits. Portfolio problem 12.7. Due on Thursday 11/15. Note it is a short homework because you have a midterm next week.

• Week 12 (11/13, 11/15)

MIDTERM ON 11/15. Covers material from Chapters 3.4,5 and sections 6.1, 6.2. It will consist of 5 or 6 questions, at least 3 of which are either taken from the homework or are small modifications of homework questions, and at least one of which will be to state and prove a theorem from the following list: Proposition 3.23, Proposition 3.31, Theorem 4.34, Theorem 5.21. Reading: Analysis notes, section on algebraic properties of limits Exercises to hand in: 12.10, 12.11 on Thursday 11/29.

• Week 13 THANKSGIVING BREAK

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• Week 14 (11/27, 11/29)
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Reading: Analysis notes, sections on series and on limits of functions

Exercises to hand in: 12.12, 12.14, 12.15, 12.18, 12.20, 12.26 on Thursday 12/6.

• Week 15 (12/4, 12/6)

Reading: None. Discussion and polishing of final version of portfolio, which must be handed in on or before Friday 12/7 NOON.