Math 2144Q - Advanced Calculus IV - Practice Final

Instructions:

- You may not refer to any notes or your textbook. No calculators, cellphones, or other electronic devices are permitted.
- You have from 12:30pm until 1:45pm to complete the test.
- In every question **you must justify your answers**.
- In any question, you may use the result of an earlier part to solve a later part, even if you did not solve the earlier part. For example, you may use the result of 3(a) to solve 3(c) even if you did not do 3(a).

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1. Let $F(x, y, z) = (3y, -xz, yz^2)$. Compute $\iint_S (\nabla \times F) \cdot dS$ where S is the surface $2z = x^2 + y^2$ below the plane $z = 2$, oriented downwards.

2. Let $f(x, y, z) = x^2 + 3y^2 + 2z^2 - 2xy + 2xz$. Find and classify all critical points of f.

3. Six students are each given a real 3×3 matrix (different students get different matrices). They compute the eigenvalues and eigenvectors, using the standard basis, and write them in a table, in which v_j is the eigenvector corresponding to λ_j. The results are:

NAME	λ_1	v_1	λ_2	v_2	λ_3	v_3
Amy	1	$(1,0,1)$	-1	$(0,1,0)$	0	$(1,1,0)$
Bob	$\frac{1}{\sqrt{2}}(1-i)$	v_1	$\frac{1}{\sqrt{2}}(1+i)$	v_2	i	v_3
Carrie	3	$(0,1,1)$	-1	$(1,1,1)$		
Devon	2	$(1,-2,0)$	2	$(0,1,-1)$	-4	$(0,0,1)$
Elizabeth	0	$(2,-1,-1)$	-1	$(-4,2,2)$	1	$(2,0,0)$
Frank	i	$(0,1,i)$	$-i$	$(0,1,-i)$	1	$(1,0,0)$

In each of the following **you must justify your answer**

(a) Which student(s) (if any) can you be sure made a mistake? (i.e. put a line in the table which is impossible.)

(b) Which student(s) (if any) were given a matrix that could be diagonalized? (Omit those listed in part a.)

(c) Which student(s) (if any) were given a unitary matrix? (Omit those listed in part a.)

(d) Write down the matrix for Amy's transformation.

4. Find the general solution of the differential equation $y''' - 4y' = 2\cosh(2x)$.

5. Solve the simultaneous differential equations $2y' - 16z + 6y = -2$ and $z' = 3z + 2y + 1$ with $y(0) = 1$, $z(0) = -1$.

6. Suppose that A is an $n \times n$ matrix having n distinct eigenvalues. Let $p(\lambda)$ be the characteristic polynomial of A. Without using the Cayley-Hamilton theorem, show that $p(A) = 0$ (note that both sides of this equation are matrices).