pg 145: 2, 8, 12

Grade 2, 8, 12

2/ Solve:
$xy' - 2y = x^5$ on $(0, \infty)$ with $y(1) = 1$.

(10 pts)

Since $x > 0$ can make $y' - 2\frac{y}{x} = x^4$] +1

and use integ factor $\exp\left(\int \frac{-2}{x}\right) = \exp(-2\log x) = \frac{1}{x^2}$, so

see $\frac{y'}{x^2} - \frac{2y}{x^3} = x^2$ is $\frac{d}{dx}\left(\frac{y}{x^2}\right) = x^2$ +4

$$\Rightarrow \frac{y}{x^2} = \frac{x^3}{3} + c$$

$$\Rightarrow y = \frac{x^5}{3} + cx^2$$ +1

Not needed for points.

[Can check satisfies D.E. b/c $y' = \frac{5x^4}{3} + 2cx$

$$\Rightarrow xy' - 2y = \frac{5x^5}{3} + \cancel{2c}x^2 - \frac{2x^5}{3} - \cancel{2c}x^2$$

$$= x^5$$]

Now condition $y(1) = 1$ says $\boxed{1 = \frac{1}{3} + c \Rightarrow c = \frac{2}{3}}$ +2 pts

So soln is $\boxed{y = \frac{1}{3}(x^5 + 2)}$ +2 pts

8) (5 pts)

Find all solns of $y'' + 4y = 0$.

It is of the form $y'' + ay' + by = 0$ for $a = 0$, $b = 4$

Let $d = a^2 - 4b = -16 < 0$, so solns are

$y = e^{-\frac{ax}{2}}[c_1 \cos kx + c_2 \sin kx]$ for $k = \frac{1}{2}\sqrt{-d}$
$= \frac{1}{2} 4 = 2$.

but $a = 0$ so

$y = c_1 \cos 2x + c_2 \sin 2x$.

Either way is 5 pts

Total (Max of 5 pts)

It is ok if did: $y'' + 4y = 0$

Char poly $r^2 + 4 = (r+2i)(r-2i)$

Solns e^{2ix}, e^{-2ix}

or $\cos 2x$, $\sin 2x$.

$\Rightarrow$ All solns $a_1 e^{i2x} + a_2 e^{-i2x}$

or equivalently $b_1 \cos 2x + b_2 \sin 2x$.

12) (10 pts)

$y'' + k^2 y = 0$ fits $y'' + ay' + by = 0$ for $a = 0$ $b = k^2$

So $d = a^2 - 4b = -4k^2$ and $k = \frac{1}{2}\sqrt{-d}$.

Solns are $y = \cos kx$, or $y = \sin kx$, or lin combs of ~~them~~ these, unless ($k = 0$ - done later)

Solve equ 2pts

1st part, $k \neq 0$ case, out of 6 pts.

Must show for $(a,b) \in \mathbb{R}^2$ and $m \in \mathbb{R}$ $\exists$ unique soln with $y(a) = b$, $y'(a) = m$. Said gen soln $y = A\cos kx + B\sin kx$

So $y' = -Ak\sin kx + Bk\cos kx$. Write as

$$\begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} y(a) \\ y'(a) \end{bmatrix} = \begin{bmatrix} \cos kx & \sin kx \\ -k\sin kx & k\cos kx \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix}$$

Give a correct argument that $\forall \begin{bmatrix} b \\ m \end{bmatrix}$ $\exists !$ soln $\begin{bmatrix} A \\ B \end{bmatrix}$, 4pts.

and note det of matrix is $k \neq 0$ so invertible. Then any $\begin{bmatrix} b \\ m \end{bmatrix}$ gives unique choice of $\begin{bmatrix} A \\ B \end{bmatrix}$.

(eg may give fmla for soln).

+1 If $k=0$ eqn is $y''=0$ solns $1, x$.
Gen soln $A+Bx$.

Then $y(a)=A+Ba$, $y'(a)=B$ +1

So given m must have $B=m$ to get $y'(a)=m$. And given b must have

+1 $A=b-Ba$ to have $y(a)=b$. So $y(a)=b$ and $y'(a)=m$ imply soln is $y(x)=b+m(x-a)$.

(So soln exists & is unique). +1

pg 154 : 1, 3, 6, 11, 15(d, f, h), 20

Grade : 3, 11, 15f

3/ $y''' + 4y'' + 4y' = 0$. Find general soln.

(5 pts) Char poly $r^3 + 4r^2 + 4r = r(r^2+4r+4) = r(r+2)^2$ +2

So factorized diff eq $D(D+2)^2 y = 0$. either one is ✓

Lin indep set of solns is $\{1, e^{-2x}, xe^{-2x}\}$ so gen. soln +2

$y = a + be^{-2x} + cxe^{-2x}$.

+1

11/ $y^{(6)} + 4y^{(4)} + 4y'' = 0$

(5 pts) Char poly $r^6 + 4r^4 + 4r^2 = r^2(r^4 + 4r^2 + 4)$

$= r^2(r^2+2)^2$

$= r^2(r+\sqrt{2}i)^2(r-\sqrt{2}i)^2$ +2

Can be written as factorization of diff eqn

Either this (up to 3 pts)

Lin indep soln set is $\{1, x, e^{\sqrt{2}ix}, xe^{\sqrt{2}ix}, e^{-\sqrt{2}ix}, xe^{-\sqrt{2}ix}\}$ +2

giving gen. soln $a_1 + a_2x + a_3e^{i\sqrt{2}x} + a_4xe^{i\sqrt{2}x} + a_5e^{-\sqrt{2}ix} + a_6xe^{-\sqrt{2}ix}$

+1

OR

OR

Alternatively lin indep soln set

$\{1, x, \cos\sqrt{2}x, \sin\sqrt{2}x, x\cos\sqrt{2}x, x\sin\sqrt{2}x\}$ +2

this (up to 3 pts)

So gen soln $y = b_1 + b_2x + b_3\cos\sqrt{2}x + b_4\sin\sqrt{2}x + b_5x\cos\sqrt{2}x + b_6x\sin\sqrt{2}x$

+1

15f/ $u_1 = e^{-2x}\cos 3x,\ u_2 = e^{-2x}\sin 3x,\ u_3 = e^{-2x},\ u_4 = xe^{-2x}$.

(5 pts) First two are killed by $(D+2+3i)(D+2-3i) = ((D+2)^2+9) = (D^2+4D+13)$ +2

u_3 killed by $(D+2)$, u_4 by $(D+2)^2$. +2 So a suitable linear de. is

$Ly = 0$ for $L = (D+2)^2(D^2+4D+13) = (D+2)^4 + 9(D+2)^2$

$= D^4 + 8D^3 + 24D^2 + 32D + 16 + 9D^2 + 36D + 36$

$= D^4 + 8D^3 + 33D^2 + 68D + 52$

equivalently, eqn is $y^{(4)} + 8y^{(3)} + 33y^{(2)} + 68y' + 52y = 0$. +1