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grade : 1, 5, 8, 11

1/ 10 pts

a) $A = A^t = A^*$ so symmetric & Hermitean. 2 pts for one, 3 for both

b) $A \neq A^t$, $A \neq A^*$, $A \neq -A^t$, $A = -(A^*)$ so skew Hermitean. 2 pts

c) $A = -A^t \neq A^t$, $A \neq A^*$, $A \neq -A^*$ so skew symmetric. 2 pts

d) $A = -A^t = -A^*$ so skew symmetric and skew Hermitean. 2 pts for one, 3 for both

5/ 10 pts

$A = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$ Find orth set evects & unitary C so $C^{-1}AC$ is diag.

Char poly is $\begin{vmatrix} \lambda-9 & -12 \\ -12 & \lambda-16 \end{vmatrix} = \lambda^2 - 25\lambda + 9(16) - 12^2 = \lambda(\lambda-25)$ so $\boxed{\text{evals } 0, 25}$ 2 pts

Evects from nullsp of $\begin{bmatrix} -9 & -12 \\ -12 & -16 \end{bmatrix}$ so $\boxed{\begin{bmatrix} 4 \\ -3 \end{bmatrix}}$ 2 pts

and of $\begin{bmatrix} 16 & -12 \\ -12 & 9 \end{bmatrix}$ so $\boxed{\begin{bmatrix} 3 \\ 4 \end{bmatrix}}$ 2 pts

They are orthogonal (as they must be b/c $A = A^*$).

Normalize by dividing by $\sqrt{4^2+3^2} = 5$ to get

$\boxed{\text{Evect } \begin{bmatrix} 4/5 \\ -3/5 \end{bmatrix} \text{ eval } 0}$ +1 pt, $\boxed{\text{Evect } \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix} \text{ eval } 25}$. +1 pt

And $C = \begin{bmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$

2 pts (Note: Order of columns not important)

8) $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$

10 pts

Char poly $\begin{vmatrix} \lambda-1 & -3 & -4 \\ -3 & \lambda-1 & 0 \\ -4 & 0 & \lambda-1 \end{vmatrix} = (\lambda-1)\begin{vmatrix} \lambda-1 & -3 \\ -3 & \lambda-1 \end{vmatrix} - 4\begin{vmatrix} -3 & -4 \\ \lambda-1 & 0 \end{vmatrix}$

$$= (\lambda-1)(\lambda^2-2\lambda+1-9) - 4(4(\lambda-1))$$

$$= (\lambda-1)(\lambda^2-2\lambda-8-16)$$

$$= (\lambda-1)(\lambda^2-2\lambda-24)$$

$$= (\lambda-1)(\lambda-6)(\lambda+4)$$

Evals $\boxed{1, 6, -4}$ 1 pt each (total of 3)

Evects :

$\lambda = 1$ $\begin{bmatrix} 0 & -3 & -4 \\ -3 & 0 & 0 \\ -4 & 0 & 0 \end{bmatrix}$ just $\begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix}$ +1 pt

$\lambda = 6$ $\begin{bmatrix} 5 & -3 & -4 \\ -3 & 5 & 0 \\ -4 & 0 & 5 \end{bmatrix}$ get $\begin{bmatrix} 1 \\ 3/5 \\ 4/5 \end{bmatrix}$ +1 pt

$\lambda = -4$ $\begin{bmatrix} -5 & -3 & -4 \\ -3 & -5 & 0 \\ -4 & 0 & -5 \end{bmatrix}$ get $\begin{bmatrix} 1 \\ -3/5 \\ -4/5 \end{bmatrix}$ +1 pt

Easy to check are orthog (not necessary b/c must be - it is just a check on alg errors)

Normalize :

Eval 1 , Evect $\frac{1}{5}\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$ +1 pt

Eval 6 , Evect $\frac{1}{5\sqrt{2}}\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$ +1 pt

Eval −4 , Evect $\frac{1}{5\sqrt{2}}\begin{bmatrix} 5 \\ -3 \\ -4 \end{bmatrix}$ +1 pt

Matrix C is then

$$C = \frac{1}{5\sqrt{2}} \begin{bmatrix} 0 & 5 & 5 \\ 4\sqrt{2} & 3 & -3 \\ 3\sqrt{2} & 4 & -4 \end{bmatrix}$$ + 1 pt.

(Note order of columns not ~~[illegible]~~ important)

11/ $a \neq 0$ and $A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$

a) find ON set evects: Char poly $\lambda^2 + a^2 = 0$

Evals $\boxed{\lambda = \pm ia}$ 1 pt each (2 pts total)

Evects from $\begin{bmatrix} ia & -a \\ a & ia \end{bmatrix}$ null sps $\begin{bmatrix} i \\ -1 \end{bmatrix}$ eval ia

$\begin{bmatrix} -ia & -a \\ a & -ia \end{bmatrix}$ nullsps $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ eval $-ia$

Could put as $\begin{bmatrix} 1 \\ i \end{bmatrix}$ + 1 pts and $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ + 1 pt

eval ia eval $-ia$

Make ON, so eval ia evect $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ + 1 pt

eval $-ia$ evect $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$ + 1 pt

b) $C = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$ 2 pt

Note! (order of cols not important)

c) If there was real orthog C so $C^{-1}AC$ is diag then $C^{-1}AC$ is real b/c all matrices in product are real, but diag matrix would be $\begin{bmatrix} ia & 0 \\ 0 & -ia \end{bmatrix}$ which is not real. +2 pt pg 134 : 3, 7

grade : 7

7/ $3x_1^2 + 4x_1x_2 + 8x_1x_3 + 4x_2x_3 + 3x_3^2$

is quad form xAx^T for $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ sym matrix

1 pt

Char poly is $\begin{vmatrix} \lambda-3 & -2 & -4 \\ -2 & \lambda & -2 \\ -4 & -2 & \lambda-3 \end{vmatrix} = (\lambda-3)\begin{vmatrix} \lambda & -2 \\ -2 & \lambda-3 \end{vmatrix} + 2\begin{vmatrix} -2 & -2 \\ -4 & \lambda-3 \end{vmatrix}$

$- 4\begin{vmatrix} -2 & \lambda \\ -4 & -2 \end{vmatrix}$

$= (\lambda-3)(\lambda^2-3\lambda-4) + 2(-2\lambda+6-8) - 4(4+4\lambda)$

$= (\lambda-3)(\lambda-4)(\lambda+1) - 4(\lambda+1) - 16(\lambda+1)$

$= (\lambda+1)(\lambda^2-7\lambda+12-20)$

$= (\lambda+1)(\lambda^2-7\lambda-8)$

$= (\lambda+1)(\lambda+1)(\lambda-8)$

Evals $-1, 8$. + 2 pts

Two lin indep +2 pt

Evects from nullsps of $\begin{bmatrix} -4 & -2 & -4 \\ -2 & -1 & -2 \\ -4 & -2 & -4 \end{bmatrix}$ so $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$ (eg).

and of $\begin{bmatrix} 5 & -2 & -4 \\ -2 & 8 & -2 \\ -4 & -2 & 5 \end{bmatrix}$ so $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ + 1 pt

Now with eval 8, can only have $\frac{1}{3}\begin{bmatrix}2\\1\\2\end{bmatrix}$ (or its -ve)

Normalize +1

But with eval -1 the esps is 2 dim, so any ON basis of this 2-dim sps is ok.

+1 pt

eg $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix}$ +1 pt and $\frac{1}{3\sqrt{2}}\begin{bmatrix}1\\-4\\1\end{bmatrix}$ works.

Normalize

This one would give orthogonal diagonalizing matrix

$$C = \frac{1}{3\sqrt{2}}\begin{bmatrix}2\sqrt{2} & \sqrt{2}\cdot 3 & \sqrt{2}\\ 1 & 0 & -4\sqrt{2} \\ 2 & -\sqrt{2} & \sqrt{2}\end{bmatrix}$$

Put in columns (any order) +1

but can have others corresp to other choices of ON basis for the -1 esps.

(Note: If had different ON basis for -1 esps then C will be different, but consistent).