Math 2142 Midterm: Things you should definitely know.

Note: This is not an exhaustive list of topics we covered in class, but if you have a solid grasp of these topics and can do problems with them you should be fine on the midterm.

1. Existence and uniqueness of solutions for first order linear IVPs $y' + P(x)y = Q(x)$, $y(a) = b$, and how to solve them by the integrating factor method (Theorem 8.3 on page 310)
2. Existence and uniqueness of solutions for second order constant coefficient linear IVPs $y''+ay'+by = R(x)$, $y(a) = b$, $y'(a) = c$, and how to solve them using general solution of homogeneous equation (Theorem 8.7 page 326) and a particular solution (Theorem 8.8 page 330). Also how to obtain particular solution using Wronskian method (Theorem 8.9 page 330) and various "guessing" methods (Section 8.16).
3. Concept of integral curve and direction field (pages 341 and 343) and how to draw them, especially how to draw direction field from equation and obtain rough sketch of integral curves from direction field.
4. How to recognize and solve first order separable equations (Theorem 8.10 page 345), including how to determine the domain.
5. How to recognize and solve first order homogeneous equations by a substitution $y = xv$ or $y = \frac{x}{v}$. Including how homogeneity is seen on direction field (Section 8.25).
6. The arithmetic of complex numbers. Field properties (Sections 9.2 and 9.4)
7. Representing complex numbers on the complex plane. (page 363)
8. Modulus and argument of a complex number, complex numbers in polar co-ordinates. Conversion from rectangular $x + iy$ co-ordinates to polar co-ordinates and vice-versa. (Section 9.5)
9. The complex exponential and writing polar co-ordinates as $re^{i\theta}$ for complex numbers. The use of this version of polar form for problems involving products, quotients and roots. (end of page 367, beginning of 368.)
10. Definition of derivatives and integrals of complex-valued functions (page 369) and Theorems 9.5 and 9.6 on page 370.
11. Definition of limit of sequence on page 379. Computation of limits from definition.
12. Monotonic bounded sequences converge (Theorem 10.1 page 381).
13. Definition of convergent series (page 384). Must be confident at converting convergence problem into limit of partial sums.

14. This is not specifically in the book, but you should understand how convergence of a sequence or a series depends only on the "tail", meaning that the beginning of the sequence or series is not relevant. You should be able to make this precise in practice, for example by showing that convergence of $\sum_N^\infty a_n$ is equivalent to convergence of $\sum_1^\infty a_n$.

15. Linearity of convergent series, Theorem 10.2 on page 385, and its application to proving divergence in Theorem 10.3.

16. Telescoping series (Section 10.7). Recognizing these (sometimes you can spot it by computing the first few terms and seeing the cancelation happen), and computing the sum of series of this type (Theorem 10.4 page 386).

17. Geometric series (Section 10.8). Recognizing these, knowing when they are convergent and divergent, and computing sum of such series (see Theorem 10.5).

18. Terms converging to zero are a necessary but not sufficient condition for convergence of series (Theorem 10.6). Should know both how to use this to prove a series diverges, and an example which shows the condition is not sufficient for convergence.

19. Comparison tests (regular and limit comparison, Theorem 10.7, and especially Theorems 10.8 and 10.9). You should know not only what these say (including the conditions you need to check in order to apply them), but should also be proficient at using them in problems. You should also know some series to test/compare against, such as the geometric series and those in Examples 1 and 2 on page 398.

20. Integral test (Theorem 10.11 page 397). You should know the conditions needed to apply this, how the proof is connected to pictures of the integrals (page 397) and know how to apply this theorem in problems. A useful rule of thumb is that if you have a series for which the n^{th} terms does not "look like" a geometric series term r^n (so root and ratio tests fail), and is not a simple comparison with n^{-s} for some $s > 0$, then you should think of applying the integral test. Don't forget to check that the function is positive and decreasing, at least for large values of n.

21. The root and ratio tests (Theorems 10.12 and 10.13 on page 400). You should know what these say about when a series converges or diverges, including the conditions under which they can be applied. You should also know what they don't say, in the sense of when they are inconclusive, including examples that demonstrate that they are inconclusive in these cases. You should have at least an intuitive understanding that they are ways of comparing the given series to a geometric series.

22. You should know the Leibniz rule (Theorem 10.14 on page 404) and how to apply it in problems, as well as have some idea about the picture (page 404) which explains the proof.

23. You should know the definition of absolute convergence and conditional convergence (page 406), and Theorem 10.15 on page 406 which says that absolute convergence implies convergence. In particular you should be comfortable with applying convergence tests for positive series (eg comparison, root, ratio, integral tests) to the absolute values of terms from a non-positive sequence to prove absolute convergence.

24. You should be able to recognize both types of improper integrals and write down what it means for an improper integral to converge. Take special care with integrals that have two or more points where they are improper, or that are improper at two sides of a finite point.

25. You should be comfortable with using comparison on integrals with positive functions, as in Theorems 10.23, 10.24 and 10.25 on page 418. You should also be able to do comparison on absolute values of non-positive functions (eg complex-valued functions) to prove convergence of improper integrals.

26. You should have a firm grasp on how the convergence or divergence of $\int_1^\infty f(x)\,dx$ and $\sum_n f(n)$ are related. As importantly, you should know how they are not related. You should know examples that show the need for f to be positive, and examples that show the need for f to be decreasing. Some such properties are explored in Exercises 20–25 on page 421.