Math 2110 Practice Final

Instructions: You may not refer to any notes or your textbook. No calculators are permitted. You have from 3:30pm until 5:30pm to complete the test.

1. Suppose that $f(x, y) = x^3y + xy^2 - y$, but $x(s, t) = s\cos t$ and $y(s, t) = ts$. Compute $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ at $(s, t) = (3, \pi)$.

2. Set up two iterated integrals (in different orders) for the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 0, 1)$, $(1, 4, 0)$. **(DO NOT EVALUATE THESE INTEGRALS.)**

3. Find the distance from $(8, 2, 3)$ to the plane $x + 2y - 2z = 7$.

4. Compute
$$\iint_R \tfrac{3}{2} e^{(y^{3/2})} dA$$
where R is the region between the curves $x = 0$, $y = x^2$ and $y = 4$. (Hint: The order of integration may affect whether you can do the integral!)

5. Find all local maximum, minimum and saddle points of
$$f(x, y) = 4x^3 + xy^2 + 2x^2y - 9x$$

6. Evaluate
$$\int \left(y + 7e^{\sqrt{x}}\right) dx + \left(3x + 7\cos(y^2)\right) dy$$
over the positively oriented boundary of the region between $y = x^2$ and $y^2 = x$.

7. Compute $\iiint_E \sqrt{x^2 + y^2}\, dV$ where E is the region between $z = 45 - 4x^2 - 4y^2$ and $z = x^2 + y^2$.

Formula Sheet: Math 2110

- Projections of **b** onto **a**

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$
$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}$$

- Cross product of $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Theorem on cross and dot products of vectors **a**, **b**, **c**.

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$
$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

- Volume of parallelpiped determined by **a**, **b**, and **c** is $|\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}|$.
- Derivatives of dot and cross products

$$\frac{d}{dt}(\mathbf{u}(t)\cdot\mathbf{v}(t)) = \mathbf{u}'(t)\cdot\mathbf{v}(t) + \mathbf{u}(t)\cdot\mathbf{v}'(t)$$
$$\frac{d}{dt}(\mathbf{u}(t)\times\mathbf{v}(t)) = \mathbf{u}'(t)\times\mathbf{v}(t) + \mathbf{u}(t)\times\mathbf{v}'(t)$$

- Curvature is given by

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

- The binormal is $\mathbf{B} = \mathbf{T}\times\mathbf{N}$.
- Acceleration is $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ where

$$a_T = \frac{dv}{dt} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$
$$a_N = \kappa v^2 = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

- The discriminant in the second derivative test for two-variable functions is given by

$$D = f_{xx}(a,b)f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$$