MATH 1070Q
Section F.3: Present Value of Annuities and Amortization

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1. Understand when we are interested in the present value of annuities.

2. Understand how to compute the present value of annuities.

3. Understand how the present value of annuities helps us compute loan re-payments.

Present value of annuities

Suppose we want to have enough money in an account to make a payment of \$900 each quarter for the next 5 years. Our payments would total $900 \times 4 \times 5 = 18000$, but we don't need to deposit that amount right now, since we don't need to pay it in full right now. We can deposit a smaller amount, and let interest make up the difference over time.

If our account has an annual rate of 7% compounded quarterly, what single amount should we deposit now, to be able to make those payments for the next 5 years? This is the **present value** of our annuities. We have

$$PV = PMT \times \frac{1 - (1 + i)^{-n}}{i}$$

so $PV = 500 \times \dfrac{1 - (1 + \frac{0.07}{4})^{-4(5)}}{0.07} = 15077.59$ (2 dec plc)

Lottery winnings

Suppose we win the lottery, and we have the option of receiving a lump sum of $1,500,000 right now, or payments of $85,000 each year for the next 33 years. Assume interest rates are 5.2% for the next 33 years. Which option should we choose?

Key idea: money increases in value over time (if we invest it). So we have to compare the full values of our two options at the same time. We know the value of the lump sum is $1,500,000 right now, and we can compare this to the present value of the annuities.

Amortizing a loan

Amortizing a loan means paying off the loan. This is normally done in regular payments at regular intervals (ie. annuities) and we can treat the value of the loan as the present value of the annuities.

For example, suppose we need to amortize (pay off) a $50,000 loan over 20 years at an annual rate of 4.7% compounded monthly. What will our monthly payments be?

1. The present value of annuities is what the annuities would be worth right now, taking interest into account.

2. We can compute the present value of annuities using the formula

$$PV = PMT \times \frac{1 - (1 + i)^{-n}}{i}$$

3. We can use the present value of annuities formula to find the monthly payments necessary to amortize a loan.