MATH 1070Q

Section 5.1: The Multiplication Principle and Permutations

Myron Minn-Thu-Aye

University of Connecticut

Objectives

1. Understand and be able to use the multiplication principle.
2. Understand what a permutation is and how to identify and count permutations.

An experiment, and the multiplication principle

We toss a coin, then draw a card from a standard deck. What is the size of the sample space? (This is useful if we want to talk about probability).

Each element of the sample space consists of either heads or tails, together with 1 of the 52 cards in a standard deck.

We have 2 choices for whether the coin is showing heads or tails, then 52 choices for the card drawn. This gives a total of

$$2 \times 52 = 104$$

possible outcomes in the sample space!

This is an example of the **multiplication principle:** in a sequence of two tasks, if there are n_1 ways to perform Task 1, and n_2 ways to perform Task 2, then the total number of ways to perform both tasks is $n_1 n_2$.

Selecting officers for a committee

A committee of 10 people needs to elect a chair and a secretary (and they must be different people). How many ways can this be done?

• Make selecting the chair our first task, with 10 ways to do it.
• Then there are 9 ways to complete our second task, selecting the secretary (the person selected as chair cannot also be the secretary).
• Using the multiplication principle, there are $10 \times 9 = 90$ total ways.

What if the committee consists of 6 women and 4 men, and the chair and secretary must be of different genders?

Factorial notation

In an election with 4 candidates, voters must rank all candidates in order of preference from 1st to 4th. How many possible rankings are there?

If n is a positive whole number, "n factorial" is

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1.$$

Example: We need to visit 6 different stores to run errands. How many different orders can we visit the stores in?

Permutations

A **permutation** is an ordering of a set of objects.

Example: In a race with 10 runners, medals will be given for 1st, 2nd, and 3rd place. How many different top threes can there be?

This can also be written as $P(10, 3)$—the number of permutations of 3 elements from a set of 10 elements.

In general, the formula for permutations is

$$P(n, r) = \frac{n!}{(n-r)!}.$$

Check this for our example above: $P(10, 3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720.$

Arranging people for a photo

A family consisting of two parents and three children are taking a photograph. If the parents must be next to each other, how many ways are there to arrange the five family members?

Recap

1. The multiplication principle allows us to count the number of ways to complete a sequence of tasks by multiplying together the number of ways to complete each task.
2. A permutation is a specific ordering of some objects.
3. We can use factorial notation ($n!$) and permutation notation ($P(n, r)$) to describe calculations involved in counting permutations.