MATH 1070Q

Section 4.2: The Number of Elements in a Set

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Objectives

1. Recognize when we are counting elements of sets that intersect
2. Develop and use formulas and/or Venn diagrams for these exercises.

Counting elements in two sets that intersect

A firm produced 100 watches, but only 90 had batteries installed, 84 had second hands installed, and 77 had both batteries and a second hand. How many of the watches had a battery, or a second hand, or both?

Let B be the set of watches with batteries.
Let S be the set of watches with second hands.

We want the number of elements in the set $B \cup S$, denoted $n(B \cup S)$.

$n(B) + n(S)$ counts $n(B \cap S)$ twice.

$$\text{So } n(B \cup S) = n(B) + n(S) - n(B \cap S)$$
$$= 90 + 84 - 77 = 97$$

In general, for any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Counting double majors

In a survey of students at the university, it is found that 371 students are majoring in chemistry, 228 students are majoring in history, and 575 are majoring in chemistry or history (or both). How many students are majoring in both chemistry and history?

Let C be the set of students majoring in chemistry.
Let H be the set of students majoring in history.
We want to find $n(C \cap H)$.

(a) Method 1 - use our formula:

Counting double majors (cont.)

(b) Method 2 - use a Venn diagram:

(c) How many students are majoring in history but not chemistry?

Counting elements in three sets that intersect

In a survey of 1000 people found that in the past month, 500 had been to Burger King, 700 to McDonald's, 400 to Wendy's, 300 to Burger King and McDonald's, 250 to McDonald's and Wendy's, 220 to Burger King and Wendy's, and 100 to all three.

(a) How many had been to Wendy's only?

(b) How many had been to Burger King or McDonald's but not both?

(c) How many had been to none of the three?

Three sets again

Suppose $n(U) = 250$, $n(A) = 130$, $n(B) = 55$, $n(C) = 90$, $n(A \cap B) = 20$, $n(A \cap C) = 35$, $n(B \cap C) = 15$, and $n(A \cap B^C \cap C) = 25$.
(a) Find $n(B \cap A^C)$
(b) Find $n((A \cup C) \cap B^C)$

Recap

1. When counting elements in sets that intersect, we must be careful to only count each element once.
2. The formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ describes how to count elements in two sets that intersect.
3. We can also use Venn diagrams to help us count elements in sets, and they are especially useful when we are considering more than two sets.