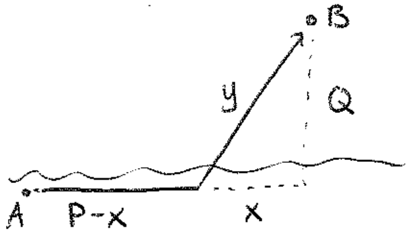
MATH 1550 Homework 6

1)



P and Q are constants.

Constraint: $x^2 + Q^2 = y^2$

$\sqrt{x^2+Q^2} = y$

Minimize time $= T = \dfrac{P-x}{2} + \dfrac{y}{1}$

time to run P-x meters at 2m/s

time to swim y meters at 1 m/s.

$$T = \frac{P}{2} - \frac{x}{2} + \underbrace{\sqrt{x^2+Q^2}}$$

differentiate using chain rule:

Let $u = \sqrt{x^2+Q^2}$, $t = x^2+Q^2$

$u = t^{\frac{1}{2}}$ $\quad \dfrac{dt}{dx} = 2x$

$\dfrac{du}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$

$\dfrac{du}{dx} = \dfrac{du}{dt} \cdot \dfrac{dt}{dx} = xt^{-\frac{1}{2}} = \dfrac{x}{\sqrt{x^2+Q^2}}$

$$\frac{dT}{dx} = 0 - \frac{1}{2} + \frac{x}{\sqrt{x^2+Q^2}} = 0$$

$$\frac{x}{\sqrt{x^2+Q^2}} = \frac{1}{2}$$

$$\frac{x^2}{x^2+Q^2} = \frac{1}{4}$$

$$4x^2 = x^2+Q^2$$

$$3x^2 = Q^2$$

$$x^2 = \frac{Q^2}{3}$$

$$x = \frac{Q}{\sqrt{3}}$$

The lifeguard should run $\underline{\underline{P - \dfrac{Q}{\sqrt{3}}}}$ meters, then jump in

2) Let $f(x)$ be continuous on $[a,b]$ and let $A(x) = \int_a^x f(t)\,dt$.

Then $A'(x) = \frac{d}{dx} \int_a^x f(t)\,dt = f(x)$.

Let $f(x)$ be continuous on $[a,b]$ and let $F(x)$ be any antiderivative of $f(x)$ on $[a,b]$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$.

3) $A(x) = \int_3^x 2t^2 - t - 21\,dt$

$A'(x) = 2x^2 - x - 21$ by FTCI.

$2x^2 - x - 21 = 0$

$(2x-7)(x+3) = 0$

$x = \frac{7}{2},\ x = -3.$

$A''(x) = 4x - 1.$

$A''(\frac{7}{2}) = 4(\frac{7}{2}) - 1 = 13 > 0$ so $x = \frac{7}{2}$ is a <u>local min</u>

$A''(-3) = 4(-3) - 1 = -13 < 0$ so $x = -3$ is a <u>local max</u>

4) Let $y = \int_{-9}^{\csc x} \frac{7}{t} - \sin^2 t + e^{2t-4}\,dt$

Let $v = \csc x$ so $y = \int_{-9}^{v} \frac{7}{t} - \sin^2 t + e^{2t-4}\,dt$

$\frac{dv}{dx} = -\csc x \cot x \qquad \frac{dy}{dv} = \frac{7}{v} - \sin^2 v + e^{2v-4}$ (FTCI)

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx} = \left(\frac{7}{\csc x} - \sin^2(\csc x) + e^{2\csc x - 4}\right)(-\csc x \cot x)$$

5) $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sec^2(2x) + \frac{5}{2x} - 7x^2\,dx = \left[\frac{1}{2}\tan(2x) + \frac{5}{2}\ln|x| - \frac{7}{3}x^3\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$

$\boxed{\text{BUT } \sec^2(2\cdot\frac{\pi}{4}) \text{ is undefined}}$

$\boxed{\text{BUT } \tan(2\cdot\frac{\pi}{4}) \text{ is undefined}}$

6) $\int \frac{1}{4x(\ln x)^3}\,dx$ Let $u = \ln x$ so $\frac{du}{dx} = \frac{1}{x} \Rightarrow x\,du = dx$

$= \int \frac{1}{4x\,u^3}\, x\,du$

$= \int \frac{1}{4} u^{-3}\,du$

$= -\frac{1}{8}u^{-2} + C$

$= \underline{\underline{-\frac{1}{8}(\ln x)^{-2} + C}}$