MATH 1550 Homework 5

1) $f(x) = \frac{3}{50}\left(\frac{1}{4}x^4 - 3x^3 + \frac{15}{2}x^2 + 9x - 10\right)$

slope $= f'(x) = \frac{3}{50}(x^3 - 9x^2 + 15x + 9)$ ← maximize/minimize on $[0,7]$

$f''(x) = \frac{3}{50}(3x^2 - 18x + 15) = 0$ find critical pts

$$\frac{9}{50}(x^2 - 6x + 5) = 0$$

$$\frac{9}{50}(x-1)(x-5) = 0$$

$$x = 1 \qquad x = 5$$

$f'(0) = \frac{3}{50}(9) = \frac{27}{50}$

$f'(1) = \frac{3}{50}(1 - 9 + 15 + 9) = \frac{48}{50}$ max

$f'(5) = \frac{3}{50}(125 - 225 + 75 + 9) = -\frac{48}{50}$ min

$f'(7) = \frac{3}{50}(343 - 441 + 105 + 9) = \frac{48}{50}$ max

} The slope is always between -1 and 1 on $[0,7]$.

2) $f(x) = \ln(x^3 - 5x^2 + 3x - 6)$

Let $y = \ln(x^3 - 5x^2 + 3x - 6)$

$t = x^3 - 5x^2 + 3x - 6, \quad y = \ln t$

$\frac{dt}{dx} = 3x^2 - 10x + 3 \qquad \frac{dy}{dt} = \frac{1}{t}$

$f'(x) = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{3x^2 - 10x + 3}{x^3 - 5x^2 + 3x - 6} = 0$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3} \qquad x = 3$$

First derivative test:

test pt	$\frac{1}{4}$	1	4
$f'(x)$	$\frac{(\frac{3}{4} - 1)(\frac{1}{4} - 3)}{\frac{1}{64} - \frac{20}{64} + \frac{48}{64} - \frac{384}{64}}$	$\frac{(3-1)(1-3)}{1 - 5 + 3 - 6}$	$\frac{(12-1)(4-3)}{64 - 80 + 12 - 6}$
	$\frac{(-)(-)}{(-)} = (-)$	$\frac{(+)(-)}{(-)} = (+)$	$\frac{(+)(+)}{(-)} = (-)$

$x = \frac{1}{3}$ local min

$x = 3$ local max

3) $f(x) = 3x^5 - 10x^4 - 80x^3 + 17x$

$f'(x) = 15x^4 - 40x^3 - 240x^2 + 17$

$f''(x) = 60x^3 - 120x^2 - 480x = 0$

$60x(x^2 - 2x - 8) = 0$

$60x(x+2)(x-4) = 0$

$x = 0,\ x = -2,\ x = 4$

test pt	-1	-3	1	5
$f''(x)$	$60(-1)(1)(-5)$ $+$	$60(-3)(-1)(-7)$ $-$	$60(1)(3)(-3)$ $-$	$60(5)(7)(1)$ $+$

our of order

$\underline{x = 0, -2, 4}$ are all pts of inflection

4) $x = 0$: $\dfrac{0^3}{\tan 0} = \dfrac{0}{0}$ indet. form

$$\lim_{x \to 0} \frac{x^3}{\tan x} \overset{\text{L'Hopital}}{=} \lim_{x \to 0} \frac{3x^2}{\sec^2 x} = \frac{3(0)}{1^2} = \underline{0}$$

5) $x = 0$: $\dfrac{0 \sin 0}{e^0 - 0 - 1} = \dfrac{0}{0}$ indet. form

$$\lim_{x \to 0} \frac{x \sin x}{e^{2x} - 2x - 1} \overset{\text{L'Hopital}}{=} \lim_{x \to 0} \frac{\sin x + x \cos x}{2e^{2x} - 2}$$ — product rule

$x = 0$: $\dfrac{\sin 0 + 0 \cos 0}{2e^0 - 2} = \dfrac{0}{0}$

$$= \lim_{x \to 0} \frac{\cos x + \cos x - x \sin x}{4e^{2x}}$$ — product rule

$$= \frac{\cos 0 + \cos 0 - 0 \sin 0}{4e^0}$$

$$= \frac{2}{4} = \underline{\frac{1}{2}}$$