MATH 1550 Homework 4

1) $\ln(x^3 \sin y) + \cot y - 9e^{5x} = 24$

chain rule: let $v = \ln(x^3 \sin y)$, $t = x^3 \sin y$

$v = \ln t$ $\qquad \frac{dt}{dx} = 3x^2 \sin y + x^3 \cos y \frac{dy}{dx}$ (product rule.)

$\frac{dv}{dt} = \frac{1}{t}$

$$\frac{dv}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx} = \frac{3x^2 \sin y + x^3 \cos y \frac{dy}{dx}}{x^3 \sin y}$$

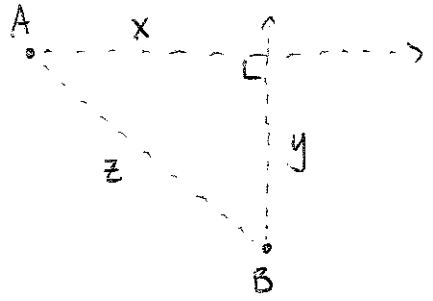
$$\frac{3x^2 \sin y + x^3 \cos y \frac{dy}{dx}}{x^3 \sin y} - \csc^2 y \frac{dy}{dx} - 45e^{5x} = 0$$

$$\frac{3}{x} + \cot y \frac{dy}{dx} - \csc^2 y \frac{dy}{dx} = 45e^{5x}$$

$$\frac{dy}{dx}(\cot y - \csc^2 y) = 45e^{5x} - \frac{3}{x}$$

$$\frac{dy}{dx} = \frac{45e^{5x} - \frac{3}{x}}{\cot y - \csc^2 y}$$

2)



$\frac{dx}{dt} = -30$ mi/hr $\qquad \frac{dy}{dt} = -20$ mi/hr

$x = 120 - 30t$ $\qquad y = 100 - 20t$

$$z^2 = x^2 + y^2$$

$$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$t = 3$: $x = 120 - 90 = 30$, $y = 100 - 60 = 40$

$$z = \sqrt{30^2 + 40^2} = \sqrt{2500} = 50$$

$$2(50)\frac{dz}{dt} = 2(30)(-30) + 2(40)(-20)$$

$$100\frac{dz}{dt} = -1800 - 1600$$

$$\frac{dz}{dt} = -\frac{3400}{100} = -34 \text{ mi/hr}$$

3) $f(x) = e^x(3x^2 - x - 1)$

$f'(x) = e^x(3x^2 - x - 1) + e^x(6x - 1)$ [product rule]

$= e^x(3x^2 + 5x - 2) = 0$

$e^x(3x - 1)(x + 2) = 0$

$x = \frac{1}{3}$ $\quad x = -2$

not in $[-3, 0]$

$f(-3) = e^{-3}(18 + 3 - 1) = 20e^{-3} = \frac{20}{e^3}$

$f(-2) = e^{-2}(12 + 2 - 1) = 13e^{-2} = \frac{13}{e^2} = \frac{13e}{e^3}$ ← MAX

$f(0) = e^0(0 - 0 - 1) = -1$ ← MIN