Homework 3

1) $y = \sqrt{e^x \sin x} = f(x)$

$t = e^x \sin x$ so $y = \sqrt{t} = t^{\frac{1}{2}}$

$\frac{dt}{dx} = e^x \sin x + e^x \cos x$ (product rule) $\quad \frac{dy}{dt} = \frac{1}{2} t^{-\frac{1}{2}}$

$f'(x) = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{2} t^{-\frac{1}{2}} (e^x \sin x + e^x \cos x)$

$= \frac{1}{2} (e^x \sin x)^{-\frac{1}{2}} (e^x \sin x + e^x \cos x)$

2) $f(x) = \frac{2x^2 - 6}{\cos(3x)}$

$f'(x) = \frac{(4x)\cos(3x) - (2x^2 - 6)(-3\sin(3x))}{\cos^2(3x)}$ [quotient rule]

3) $y = 8^{3^x} = f(x)$

$t = 3^x$ so $y = 8^t$

$\frac{dt}{dx} = (\ln 3) 3^x \quad \frac{dy}{dt} = (\ln 8) 8^t$

$f'(x) = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = (\ln 8) 8^t (\ln 3) 3^x$

$= (\ln 8) 8^{3^x} (\ln 3) 3^x$

4) $5^y + 4y^3 - \sec x + 3^x = 0$.

$(\ln 5)5^y \cdot \frac{dy}{dx} + 12y^2 \cdot \frac{dy}{dx} - \sec x \tan x + (\ln 3)3^x = 0$

$\frac{dy}{dx}\left((\ln 5)5^y + 12y^2\right) = \sec x \tan x - (\ln 3)3^x$

$$\frac{dy}{dx} = \frac{\sec x \tan x - (\ln 3)3^x}{(\ln 5)5^y + 12y^2}$$

5) $e^x \tan y + 5x = y$

$\underbrace{e^x \tan y + e^x \sec^2 y \cdot \frac{dy}{dx}}_{\text{product rule}} + 5 = \frac{dy}{dx}$

$e^x \sec^2 y \cdot \frac{dy}{dx} - \frac{dy}{dx} = -5 - e^x \tan y$

$\frac{dy}{dx}(e^x \sec^2 y - 1) = -5 - e^x \tan y$

$$\frac{dy}{dx} = \frac{-5 - e^x \tan y}{e^x \sec^2 y - 1}.$$

6) We want to find a point $x=a$ where the tangent line to $x^2+y^2=1$ passes through $(50,0)$.

differentiate: $2x + 2y \cdot \frac{dy}{dx} = 0$

$2y \cdot \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{y}$ at $x=a$, $a^2+y^2=1$

$y^2 = 1-a^2$

$y = -\sqrt{1-a^2}$

so at $x=a$, $\frac{dy}{dx} = \frac{-a}{-\sqrt{1-a^2}} = \frac{a}{\sqrt{1-a^2}}$

6) (cont.) The eqn of the tangent line at $x = a$ is

$$y + \sqrt{1-a^2} = \frac{a}{\sqrt{1-a^2}}(x-a)$$

For this to pass through $(50,0)$, set $x = 50$ and $y = 0$.

$$0 + \sqrt{1-a^2} = \frac{a}{\sqrt{1-a^2}}(50-a)$$

$$\sqrt{1-a^2}^2 = a(50-a)$$

$$1 - a^2 = 50a - a^2$$

$$1 = 50a$$

$$a = \frac{1}{50}$$