Homework 2

1) A function $f(x)$ is continuous at $x=c$ if $\lim_{x\to c} f(x) = f(c)$.

2) (a) $f(x)$ has a jump discontinuity at $x=c$ if $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$ both exist and $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$.

(b) $f(x)$ has an infinite discontinuity at $x=c$ if either $\lim_{x\to c^-} f(x)$ or $\lim_{x\to c^+} f(x)$ is infinite.

3) $f(x) = \dfrac{2x^2-x-6}{x^2+3x-10}$

Vertical asymptotes can occur where
$$x^2+3x-10=0$$
$$(x+5)(x-2)=0$$
$$x=-5 \quad x=2.$$

$x=-5$: $\dfrac{50+5-6}{0} = \dfrac{49}{0}$ so $\underline{x=-5}$ is a vertical asymptote.

$x=2$: $\dfrac{8-2-6}{0} = \dfrac{0}{0}$ so $x=2$ is not a vertical asymptote.

$\lim_{x\to\infty} \dfrac{2x^2-x-6}{x^2+3x-10} = \dfrac{2}{1} = 2$ so $\underline{y=2}$ is a horizontal asymptote.

4) For instance, $f(x) = \dfrac{-6x^2}{(x+1)(x-3)}$

5) For instance, $f(x) = \dfrac{4x^2}{3x(x-2)}$

6) Let $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x(x+0)} = \underline{\underline{-\frac{1}{x^2}}}.$$

7) $f(x) = |x|$.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$, if it exists.

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0^-} \frac{|h|}{h}$$ As $h \to 0^-$, $h < 0$ so $|h| = -h$

$$= \lim_{h \to 0^-} \frac{-h}{h}$$

$$= \lim_{h \to 0^-} (-1) = -1$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{|h|}{h}$$ As $h \to 0^+$, $h > 0$ so $|h| = h$.

$$= \lim_{h \to 0^+} \frac{h}{h}$$

$$= \lim_{h \to 0^+} 1 = 1.$$

Since the one-sided limits disagree, $f'(0)$ DNE.