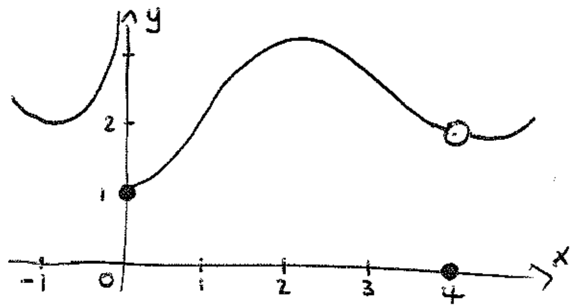
Homework 1

1) For instance



2) $f(x) = \dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x+2)(x-2)}$

Denominator $=0$ at $x=-2,2$

$x=-2$: $\dfrac{-2+2}{(0)(-4)} = \dfrac{0}{0}$ indeterminate, NOT an infinite limit

$x=2$: $\dfrac{2+2}{(4)(0)} = \dfrac{4}{0}$ we get an infinite limit.

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \dfrac{x+2}{x^2-4} = -\infty$

As $x \to 2^-$, $x<2$ so $x^2<4$ and $x^2-4<0$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \dfrac{x+2}{x^2-4} = +\infty$

As $x\to 2^+$, $x>2$ so $x^2>4$ and $x^2-4>0$.

3) $\lim\limits_{x\to 4} \dfrac{3x+1}{x^2-16}$

$x=4$: $\dfrac{12+1}{16-16} = \dfrac{13}{0}$ limit is infinite and DNE.

4) $\lim\limits_{x \to -1} \dfrac{2x^2 - x - 3}{x+1}$

$x = -1: \quad \dfrac{2+1-3}{-1+1} = \dfrac{0}{0}$ indeterminate.

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1} = \lim_{x \to -1} \frac{(x+1)(2x-3)}{x+1} = \lim_{x \to -1} (2x+3) = -2+3 = \underline{\underline{1}}$$

5) $\lim\limits_{x \to 5} \dfrac{\sqrt{x+11} - 4}{x-5}$

$x = 5: \quad \dfrac{\sqrt{16} - 4}{5-5} = \dfrac{0}{0}$ indeterminate.

$$\lim_{x \to 5} \frac{\sqrt{x+11} - 4}{x-5} = \lim_{x \to 5} \frac{\sqrt{x+11} - 4}{x-5} \cdot \frac{\sqrt{x+11} + 4}{\sqrt{x+11} + 4}$$

$$= \lim_{x \to 5} \frac{x+11-16}{(x-5)(\sqrt{x+11} + 4)}$$

$$= \lim_{x \to 5} \frac{x-5}{(x-5)(\sqrt{x+11} + 4)}$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x+11} + 4}$$

$$= \frac{1}{\sqrt{16} + 4}$$

$$= \underline{\underline{\frac{1}{8}}}$$