

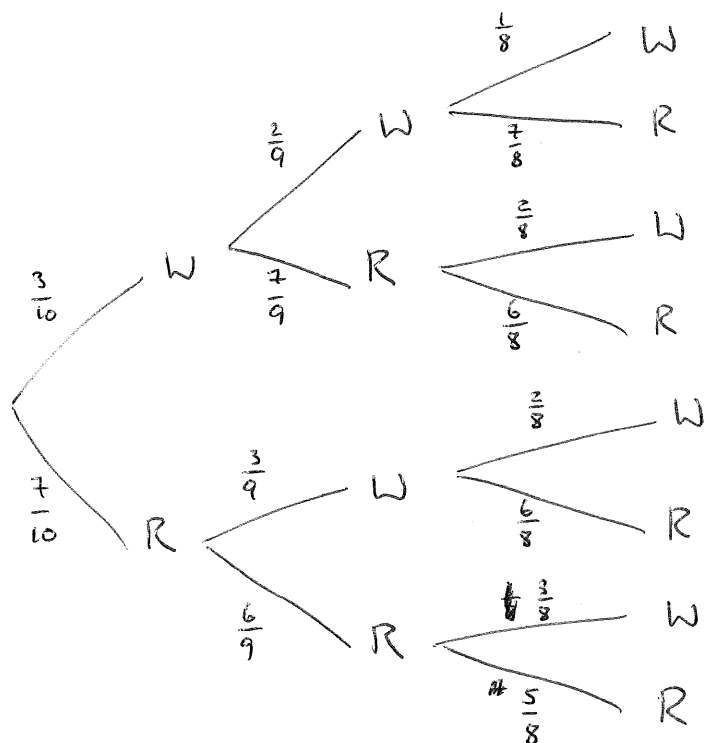


Exam 2 will cover sections 4.6, 4.7, 5.2, 5.3, 5.4, 6.1, 6.2, 6.3, 6.4, F.1, F.2, F.3 and F.4. This sample exam is intended to be used as one of several resources to help you prepare. The coverage of topics is not exhaustive, and you should look through all examples from lectures (or videos), quizzes, and homework as these will all be relevant. The wealth of problems in our text is also a good resource for practice with this material.

- The exam is a closed notes, closed book exam. You can not receive aid on this exam from anyone. Approved calculators are allowed, but there is no sharing of calculators!
- Some partial credit may be given depending on the correctness of the work submitted. You must show all work and calculations needed to reach your answers. Just using a calculator is not sufficient for credit.
- Please make sure to attend the exam that you signed up for at the beginning of the term. The room that your exam is in can be found on the common course webpage.
- Please note that the table on the last page of the sample exam will also be included on the exam.

1. Three balls are randomly drawn (without replacement) from an urn that contains three white and seven red balls.

(a) Draw a tree diagram and indicate the correct probabilities.



(b) What is the probability of drawing a white ball on the third draw?

$$P(\text{Won 3rd}) = \left(\frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8}\right) + \left(\frac{3}{10} \cdot \frac{7}{9} \cdot \frac{2}{8}\right) + \left(\frac{7}{10} \cdot \frac{3}{9} \cdot \frac{2}{8}\right) + \left(\frac{7}{10} \cdot \frac{6}{9} \cdot \frac{3}{8}\right)$$

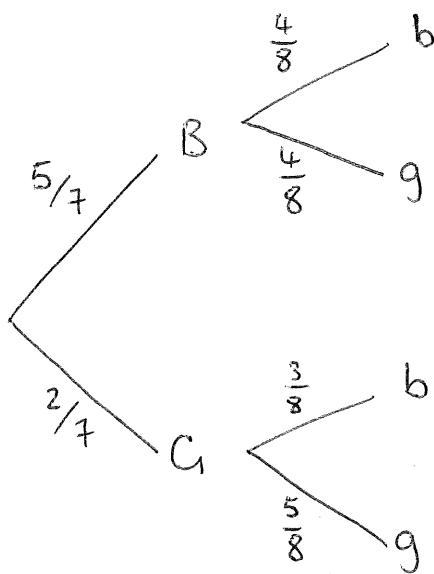
$\text{WWW} \quad \text{WRW} \quad \text{RWW} \quad \text{RRW}$

(c) What is the probability of drawing a white ball on the third draw given that at least one white ball was drawn on the first two draws?

$$P(\text{Won 3rd} \mid \text{at least 1 W on first 2}) = \frac{P(\text{Won 3rd} \cap \text{at least 1 W on first 2})}{P(\text{at least 1 W on first 2})}$$

$$= \frac{\left(\frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8}\right) + \left(\frac{3}{10} \cdot \frac{7}{9} \cdot \frac{2}{8}\right) + \left(\frac{7}{10} \cdot \frac{3}{9} \cdot \frac{2}{8}\right)}{\left(\frac{3}{10} \cdot \frac{2}{9}\right) + \left(\frac{3}{10} \cdot \frac{7}{9}\right) + \left(\frac{7}{10} \cdot \frac{3}{9}\right)}$$

2. A bag contains five blue and two green jelly beans. A box contains three blue and four green jelly beans. A jelly bean is selected at random from the bag and is placed in the box. Then a jelly bean is selected at random from the box. If a green jelly bean is selected from the box, what is the probability that the transferred jelly bean was blue?



$$P(B|g) = \frac{P(B \cap g)}{P(g)}$$

$$= \frac{\frac{5}{7} \cdot \frac{4}{8}}{\frac{5}{7} \cdot \frac{4}{8} + \frac{2}{7} \cdot \frac{5}{8}}$$

3. A basketball player makes on average 3 free throws out of every 5 attempted. If the player attempts 7 free throws, find the probability that they make at least five of them.

$$n = 7, \quad p = \frac{3}{5}, \quad q = \frac{2}{5}$$

$$P(X \geq 5) = C(7,5) \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^2$$

$$+ C(7,6) \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^1$$

$$+ C(7,7) \left(\frac{3}{5}\right)^7 \left(\frac{2}{5}\right)^0$$

4. A baseball player has a batting average of 0.250 (this is the probability of getting a hit each time they bat). The player bats 4 times in a game.

(a) What is the probability that the player gets exactly 2 hits?

$$n = 4, \quad p = 0.25, \quad q = 0.75$$

$$P(X=2) = C(4,2)(0.25)^2(0.75)^2$$

(b) What is the player's expected number of hits?

$$\begin{aligned} E(X) &= 0P(X=0) + 1P(X=1) + 2P(X=2) + 3P(X=3) + 4P(X=4) \\ &= 0 + 1C(4,1)(0.25)^1(0.75)^3 + 2C(4,2)(0.25)^2(0.75)^2 \\ &\quad + 3C(4,3)(0.25)^3(0.75)^1 + 4C(4,4)(0.25)^4(0.75)^0 \end{aligned}$$

5. If we draw a five-card hand from a standard 52-card deck, find the probability of a four of a kind and one of another kind (e.g. four Kings and one 8, or four 5s and one Ace, etc.).

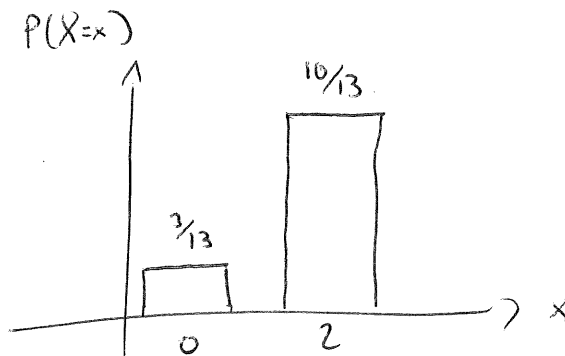
$$\frac{C(13,1) C(4,4) C(12,1) C(4,1)}{C(52,5)}$$

6. Three cards are drawn from a standard 52-card deck. What is the probability that at least two hearts are drawn?

$$\begin{array}{l} \text{2 hearts,} \quad \swarrow \quad C(13,2) C(39,1) + C(13,3) C(39,0) \quad \nwarrow \quad \text{3 hearts,} \\ \text{1 other} \quad \text{card.} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{no other cards} \end{array}$$
$$\frac{\quad}{C(52,3)}$$

7. Three balls are selected at random from an urn that contains five yellow balls and eight red balls. Let the random variable X denote the number of yellow balls drawn times the number of red balls drawn. Draw a histogram for X . Make sure to label the axes and show all probabilities.

x	0	2
$P(X=x)$	$\frac{C(5,3)C(8,0) + C(5,0)C(8,3)}{C(13,3)}$	$\frac{C(5,2)C(8,1) + C(5,1)C(8,2)}{C(13,3)}$
	$= \frac{3}{13}$	$= \frac{10}{13}$



8. Let $S = \{94, 97, 82, 68, 74, 83, 85, 91, 77, 69\}$. Find the mean, variance, and standard deviation of the values in S .

$$\mu = \frac{94 + 97 + 82 + 68 + 74 + 83 + 85 + 91 + 77 + 69}{10} = 82$$

$$\text{Var} = \frac{(94-82)^2 + (97-82)^2 + (82-82)^2 + (68-82)^2 + (74-82)^2 + (83-82)^2 + (85-82)^2 + (91-82)^2 + (77-82)^2 + (69-82)^2}{10}$$

$$= 91.4$$

$$\sigma = \sqrt{91.4} = 9.5603 \text{ (to 4 dec plcs)}$$

must
show
formulas
for credit

9. A machine produces screws with diameters which are normally distributed. The mean diameter of a screw is 0.224 inches and the standard deviation is 0.004 inches. Quality requirements demand a screw to be rejected if the diameter is more than 0.001 inches different from the mean.

(a) Find $P(X \leq 0.221)$ where X is the diameter of the screw.

$$P(X \leq 0.221) = P\left(Z \leq \frac{0.221 - 0.224}{0.004}\right) = P(Z \leq -0.75)$$

$$= 0.2266$$

must show
conversion to
z-values

(b) What is the probability of a screw being rejected?

$$P(X \leq 0.223) + P(X > 0.225)$$

$$= P\left(Z < \frac{0.223 - 0.224}{0.004}\right) + P\left(Z > \frac{0.225 - 0.224}{0.004}\right)$$

$$= P(Z < -0.25) + P(Z > 0.25)$$

$$= 0.4013 + (1 - P(Z < 0.25))$$

$$= 0.4013 + (1 - 0.5987)$$

$$= 0.4013 + 0.4013$$

$$= 0.8026$$

10. Eighteen years ago, your aunt gave you an 18-year bond with an annual interest rate of 7.5% compounded quarterly. The bond is currently worth \$10,000. What was the price of the bond when your aunt purchased it?

$$F = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$10000 = P \left(1 + \frac{0.075}{4} \right)^{4(18)}$$

$$P = \frac{10000}{\left(1 + \frac{0.075}{4} \right)^{72}} = 5518.69 \quad (\text{to 2 dec plcs})$$

11. An individual is seeking to purchase a house in 10 years time. She wishes to save \$150,000 for a down payment on a future house. She has found an account which offers an annual interest rate of 4.5% compounded monthly.

- (a) How much should she deposit into this account each month to reach her goal of \$150,000 in 10 years?

$$FV = PMT \times \frac{(1+i)^n - 1}{i}$$

$$PMT = FV \times \frac{i}{(1+i)^n - 1} = 150000 \times \frac{\frac{0.045}{12}}{\left(1 + \frac{0.045}{12} \right)^{120} - 1}$$

$$= \frac{992.08}{\cancel{119049.60}} \quad (\text{to 2 dec plcs})$$

- (b) How much total interest does this annuity earn in 10 years?

$$\text{total deposited} = \frac{992.08}{\cancel{119049.60}} \times 120 = \$ 119049.60$$

$$\text{total interest} = 150000 - 119049.60 = 30950.40$$

12. A lottery has one \$100,000 prize, two \$25,000 prizes, three \$5,000 prizes, and ten \$500 prizes. There are 100,000 lottery tickets sold at \$2 each, and each is equally likely to win. Find the expected return on buying one lottery ticket.

X = return

X	99998	24998	4998	498	-2
$P(X=x)$	$\frac{1}{100000}$	$\frac{2}{100000}$	$\frac{3}{100000}$	$\frac{10}{100000}$	$\frac{99984}{100000}$

$$E(X) = 99998 \left(\frac{1}{100000} \right) + 24998 \left(\frac{2}{100000} \right) + 4998 \left(\frac{3}{100000} \right) + 498 \left(\frac{10}{100000} \right) - 2 \left(\frac{99984}{100000} \right)$$

must write this formula!

13. A family obtains a loan of \$30,000 to buy a new car. Suppose that the annual interest rate on the loan is 8.7%, and that the family will pay off the loan over the course of 5 years.

(a) What will the family's monthly payment be?

$$PV = PMT \times \frac{1 - (1+i)^{-n}}{i}$$

$$\begin{aligned} PMT &= PV \times \frac{i}{1 - (1+i)^{-n}} = 30000 \times \frac{\frac{0.087}{12}}{1 - \left(1 + \frac{0.087}{12}\right)^{-60}} \\ &= 618.39 \quad (\text{to 2 dec plcs}) \end{aligned}$$

(b) What is the total interest the family will pay on the loan?

$$\text{total paid} = 618.39 \times 60 = 37103.40$$

$$\text{total interest} = 37103.40 - 30000 = 7103.40$$

Simple Interest

$$F = P(1 + rt)$$

$$P = \frac{F}{1 + rt}$$

Discount Loans

$$A = P(1 - rt)$$

$$r_{eff} = \frac{r}{1 - rt}$$

Compound Interest

$$F = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$P = \frac{F}{\left(1 + \frac{r}{m}\right)^{mt}}$$

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

Ordinary Annuities

$$FV = PMT \times \frac{(1 + i)^n - 1}{i}$$

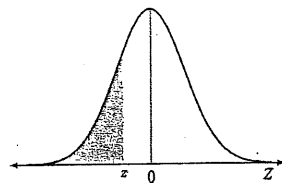
$$PMT = FV \times \frac{i}{(1 + i)^n - 1}$$

$$PV = PMT \times \frac{1 - (1 + i)^{-n}}{i}$$

$$PMT = PV \times \frac{i}{1 - (1 + i)^{-n}}$$

Area Under a Normal Curve

This table gives the area under the standard normal curve to the left of $z = \frac{x - \mu}{\sigma}$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9984	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998