



*University of Connecticut
Department of Mathematics*

MATH 1070

SAMPLE FINAL EXAM

SPRING 2016

NAME: _____

Instructor Name: _____ Section: _____

Read This First!

- This is a closed notes, closed book exam. You cannot receive aid on this exam from anyone else. Approved calculators are allowed, but there is no sharing of calculators!
 - Partial credit may be given depending on the correctness of the work submitted. You must show all work and calculations needed to reach your answers, even when using a calculator.
 - If part of your solution is written outside of the space provided, clearly indicate this.
 - A table of values for the normal distribution and a list of formulas from Chapter F can be found at the end of the exam.
 - **USE EXACT ANSWERS UNLESS OTHERWISE INDICATED.**

Grading - For Administrative Use Only

1. The supply of jet skis satisfies the equation $p = 3x + 7000$. The demand for jet skis is calculated to be 1500 when the price is \$14,250, and 3000 when the price is \$10,500.

(a) Find the demand equation.

[4]

$$m = \frac{10500 - 14250}{3000 - 1500} = -2.5$$

$$p - 14250 = -2.5(x - 1500)$$

$$p = -2.5x + 18000$$

(b) Find the equilibrium point.

[3]

$$3x + 7000 = -2.5x + 18000$$

$$5.5x = 11000$$

$$x = 2000$$

$$p = 3(2000) + 7000 = 13000$$

2. A bakery makes cupcakes and muffins. A batch of cupcakes requires 2 pounds of flour and 1 pound of sugar, while a batch of muffins requires 2.5 pounds of flour and 0.5 pounds of sugar. Each batch of cupcakes results in \$180 in revenue, and each batch of muffins results in \$160 in revenue. If the bakery has 50 pounds of flour and 13 pounds of sugar, how many batches of cupcakes and muffins should be made to maximize revenue?

(a) Write the objective function and inequalities describing this scenario.

[4]

x = batches cupcakes, y = batches muffins

$$R = 180x + 160y$$

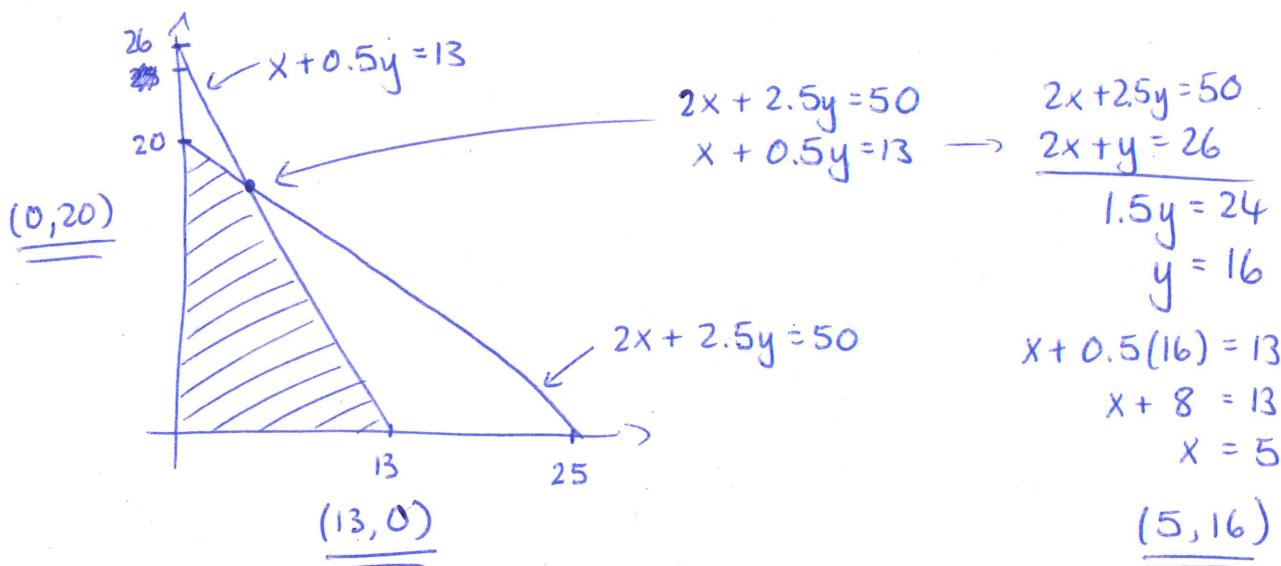
$$2x + 2.5y \leq 50$$

~~$$x + 0.5y \leq 13$$~~

$$x, y \geq 0$$

(b) Graph the feasible region and find its corner points.

[7]



(c) Determine how many batches of cupcakes and muffins should be made.

[4]

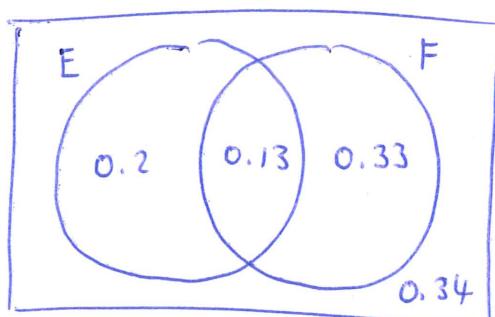
x	y	R
13	0	2340
0	20	3200
5	16	3460

5 batches cupcakes
16 batches muffins

3. Let E and F be two events with $P(E) = 0.33$, $P(F) = 0.46$, and $P(E \cup F) = 0.66$.

(a) Draw a Venn diagram of all the probabilities.

[3]



- (b) Find $P(E^c \cap F)$.

[2]

$$\underline{0.33}$$

- (c) Find $P(E|E \cup F^c)$.

[4]

$$\frac{P(E \cap (E \cup F^c))}{P(E \cup F^c)} = \frac{0.2 + 0.13}{0.2 + 0.13 + 0.34}$$

4. We toss a fair coin, then roll a fair four-sided die with faces labeled 1 through 4, then roll a fair six-sided die with faces labeled 1 through 6. What is the probability of getting tails on the coin, at least 2 on the four-sided die, and at most 2 on the six-sided die?

[4]

$$\underline{\frac{1}{2} \times \frac{3}{4} \times \frac{2}{6}}$$

5. The license plates in a country consist of a sequence of three letters followed by a sequence of four digits.

(a) How many license plates are possible if letters cannot be repeated, digits can be repeated, and the last digit cannot be the same as the first digit? [4]

$$26 \times 25 \times 24 \times 10 \times 10 \times 10 \times 9$$

- (b) How many license plates are possible if letters can be repeated, digits can be repeated, and the last digit cannot be the same as the first digit or second digit? [6]

$$(26 \times 26 \times 26 \times 10 \times 9 \times 10 \times 8) + (26 \times 26 \times 26 \times 10 \times 1 \times 10 \times 9)$$

✓
different
digits
✓
same
digit

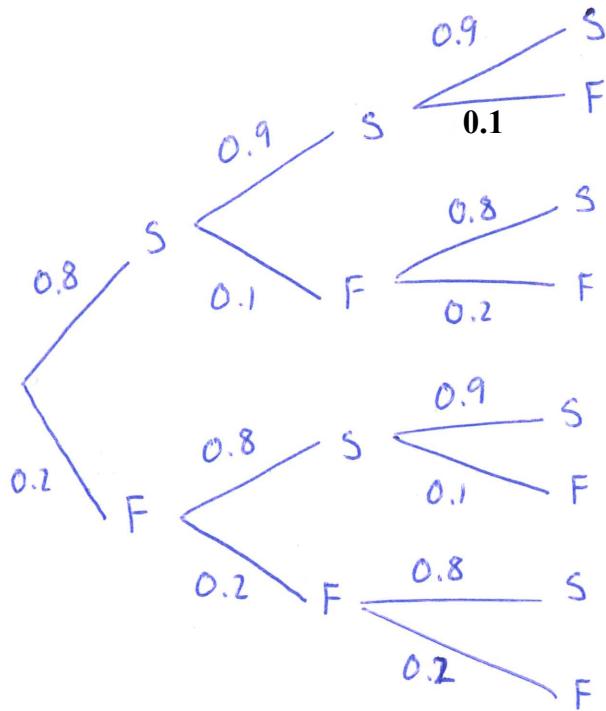
6. A soccer team is going to play six games in a tournament. The probability that the team wins each game, independent of the others, is $\frac{5}{7}$. What is the probability that the team will win at least three games in the tournament? [4]

$$C(6,3)\left(\frac{5}{7}\right)^3\left(\frac{2}{7}\right)^3 + C(6,4)\left(\frac{5}{7}\right)^4\left(\frac{2}{7}\right)^2 + C(6,5)\left(\frac{5}{7}\right)^5\left(\frac{2}{7}\right)^1 + C(6,6)\left(\frac{5}{7}\right)^6\left(\frac{2}{7}\right)^0$$

7. A basketball player attempts three free throws. The probability that she will make the first free throw is 0.8. If she makes a free throw, her confidence rises, and the probability that she will make the next free throw is 0.9. If she misses a free throw, the probability that she will make the next free throw is 0.8.

- (a) Draw a tree diagram illustrating this scenario, labeling all branches with probabilities.

[4]



- (b) What is the probability that the player makes the first free throw and the third free throw?

[3]

$$0.8(0.9)(0.9) + 0.8(0.1)(0.8)$$

SSS

SFS

- (c) If the player makes the third free throw, find the probability that she makes exactly two of the three free throws.

[4]

$$\begin{aligned}
 & \cancel{\text{Method 1}} \quad P(\text{exactly } 2 \text{ S} \mid \text{S on 3}) = \frac{P(\text{exactly } 2 \text{ S} \cap \text{S on 3})}{P(\text{S on 3})} \\
 & = \frac{0.8(0.1)(0.8) + 0.2(0.8)(0.9)}{0.8(0.9)(0.9) + 0.8(0.1)(0.8) + 0.2(0.8)(0.9) + (0.2)(0.2)(0.8)}
 \end{aligned}$$

8. A box contains 4 red balls and 2 white balls. Three balls will be drawn from the box all at once (i.e. without replacement). Let X represent the number of white balls drawn minus the number of red balls drawn.

- (a) Draw and complete a probability distribution table for X . Give exact decimal answers for all probabilities. [6]

X	-3	-1	1
$P(X=x)$	$\frac{C(2,0)C(4,3)}{C(6,3)}$	$\frac{C(2,1)C(4,2)}{C(6,3)}$	$\frac{C(2,2)C(4,1)}{C(6,3)}$
	 0.2	 0.6	 0.2

- (b) Find the expected value $E(X)$. [3]

$$E(X) = -3(0.2) - 1(0.6) + 1(0.2) = -1$$

- (c) Find the variance $\text{Var}(X)$. [3]

$$\text{Var}(X) = (-3-(-1))^2(0.2) + (-1-(-1))^2(0.6) + (1-(-1))^2(0.2) = 1.6$$

9. The heights of a population of giraffes are normally distributed with mean 192 inches and standard deviation 25 inches. Find the probability that one of these giraffes is more than 200 inches tall. [4]

$$\begin{aligned}
 P(X > 200) &= P\left(Z > \frac{200 - 192}{25}\right) \\
 &= P(Z > 0.32) \\
 &= 1 - P(Z \leq 0.32) \\
 &= 1 - 0.6255 \\
 &= 0.3745
 \end{aligned}$$

10. We are going to make a single deposit into an account with an annual interest rate of 4.5% compounded monthly, and we are going to let interest accumulate for 20 years. Then, we are going to make monthly payments of \$250 from the accumulated total for the next 10 years (assuming the same interest rate and compounding). What is the least amount of money we must deposit now in order to be able to make all of the monthly payments in the future? [7]

Last 10 yrs: $PV = PMT \times \frac{1 - (1+i)^{-n}}{i}$

$$\begin{aligned}
 &= 250 \times \frac{1 - (1 + \frac{0.045}{12})^{-12(10)}}{\frac{0.045}{12}} \\
 &= 24122.33 \leftarrow \text{how much we need at end of first 20 yrs so we can make payments for last 10 yrs.}
 \end{aligned}$$

First 20 yrs: $F = P(1+i)^n$

$$\begin{aligned}
 24122.33 &= P \left(1 + \frac{0.045}{12}\right)^{12(20)} \\
 P &= \frac{24122.33}{\left(1 + \frac{0.045}{12}\right)^{240}} = \underline{\underline{9823.93}}
 \end{aligned}$$

11. Find all solutions to the following system of equations using Gauss elimination.

[6]

$$\begin{array}{l} x + 2y + z = 3 \\ 2x + 5y - z = 11 \\ 3x + 4y + 9z = -1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & 11 \\ 3 & 4 & 9 & -1 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{array} : \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & 5 \\ 0 & -2 & 6 & -10 \end{array} \right]$$

$$2R_2 + R_3 \rightarrow R_3 : \quad \left[\begin{array}{ccc|c} x & y & z \\ 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \text{row-echelon form!}$$

↑
no leading 1 in this column,
so z can be anything
 $z = t$

~~$x = 2t - 1$~~

$$\begin{aligned} R_2 : y - 3z &= 5 \\ y - 3t &= 5 \\ y &= 5 + 3t \end{aligned}$$

$$\begin{aligned} R_1 : x + 2y + z &= 3 \\ x + 2(5 + 3t) + t &= 3 \\ x + 10t + 7t &= 3 \\ x &= \underline{\underline{-7 - 7t}} \end{aligned}$$

12. Consider the system of equations below.

$$\begin{aligned} 2x + 3y &= -4 \\ 3x + 4y &= 5 \end{aligned}$$

(a) Write the system in the form $AX = B$ for matrices A , X , and B .

[3]

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

(b) Find A^{-1} , the inverse of the matrix A from part (a). Show all row operations used.

[5]

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right]$$

$$\frac{1}{2}R_1 \rightarrow R_1: \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 3 & 4 & 0 & 1 \end{array} \right]$$

$$-3R_1 + R_2 \rightarrow R_2: \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{2} & 1 \end{array} \right]$$

$$-2R_2 \rightarrow R_2: \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 3 & -2 \end{array} \right]$$

$$-\frac{3}{2}R_2 + R_1 \rightarrow R_1: \left[\begin{array}{cc|cc} 1 & 0 & -4 & 3 \\ 0 & 1 & 3 & -2 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

(c) Solve the system using the inverse matrix A^{-1} , not by any other method.

[3]

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 + 15 \\ -12 - 10 \end{bmatrix} = \begin{bmatrix} 31 \\ -22 \end{bmatrix}$$

A^{-1}

Formulas From Chapter F

Simple Interest

$$F = P(1 + rt)$$

$$r_{\text{eff}} = \frac{r}{1 - rt}$$

Compound Interest

$$F = P \left(1 + \frac{r}{m}\right)^{mt} = P(1 + i)^n$$

$$r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 = (1 + i)^m - 1$$

Future Value of Annuities

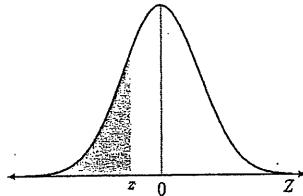
$$FV = \text{PMT} \times \frac{(1 + i)^n - 1}{i}$$

Present Value of Annuities

$$PV = \text{PMT} \times \frac{1 - (1 + i)^{-n}}{i}$$

Area Under a Normal Curve

This table gives the area under the standard normal curve to the left of $z = \frac{x-\mu}{\sigma}$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.5	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9984	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9994	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998