Introduction

My research interests are in the area of applied probability, with the main focus on mathematical finance. My results include the existence, uniqueness, stability, and asymptotic analysis of certain stochastic control and stochastic analysis problems, which are at the core of mathematical finance, see [Mos15, Mos17, CCFM17, Mos18a, LMŽ18, Mos18b, MS19, Mos20, MS20, Mos21, MSZ23, MS24a, MS24b, MM24]. Eight of my published papers are single-authored. I have been continuously supported by the National Science Foundation as a single PI since 2015. This includes the NSF CAREER award.

I try to study problems at a high, even maximal, level of generality and under minimal conditions when possible. Thus, the papers above include a range of sharp, well-posedness results and representation and asymptotic formulas in the context of classical stochastic analysis and stochastic control problems mostly related to mathematical finance. My approach to research relies on trying to understand the mathematical structure of a given problem, followed by applying appropriate mathematical techniques on the level necessary for a given problem. To find the underlying mathematical structure, I try to identify auxiliary linear and quadratic components in the context of non-linear problems via various transformations, such as changes of measure and numéraire and an increase in the dimensionality of the value functions. I also use and develop various reformulations, approximation techniques, and representation results.

The range of topics that I have been studying includes expected utility maximization, optimal consumption of multiple goods, indifference pricing in the sense of [KK21, page 157], the Epstein-Zin problem, and several topics in large markets, among others. On the probabilistic side, my results include a new characterization of stochastic dominance for the densities of equivalent martingale measures and supermartingale deflators in [MSZ23], an approximation for BSDEs with non-Lipschitz generators for stability analysis of Epstein-Zin problem in [MM24], multiplicative Doob-Meyer-Mertens-type decomposition of the dual minimizer in [MS20], a characterization of orthogonal martingales under simultaneous changes of measure and numéraire in [MS24b], among other results.

An overview of several particular research topics

There are several main directions in previous works that I discuss below. Every paper below can be downloaded from my website.

Existence and uniqueness results in stochastic control problems arising in financial mathematics and economics

The following papers address these questions in the context of the classical problems arising in mathematical finance and under weak (often weakest possible) conditions.

• In the paper "Optimal investment and consumption with labor income in incomplete markets," [MS20] co-authored with Mihai Sîrbu (UT Austin), we consider the problem of optimal consumption from labor income and investment in a general incomplete semimartingale market. The economic agent cannot borrow against future income, so the total wealth is required to be positive at (all or some) previous times. Under very general conditions, we show that an optimal consumption and investment plan exists and is unique and provide a dual characterization in terms of an optional strong supermartingale deflator and a decreasing part, which charges only the times when the no-borrowing constraint is binding. The analysis relies on the infinite-dimensional parametrization of the income/liability streams and, therefore, provides the first-order dependence of the optimal investment and consumption plans on future income/liabilities (as well as a pricing rule).

The results of this paper provide a solution to a problem that was open for about twenty five years and was studied in [HP93] and [EKJP98] in complete[1] (and Brownian) formulation. The open question was whether the solution could be obtained in incomplete settings, and we provided an affirmative answer (also in semimartingale, not just Brownian settings).

An important part of [MS20] was Theorem 6.1 on page 777, establishing a multiplicative decomposition of the dual minimizer, which is an optional strong supermartingale (thus is not necessarily left or right-continuous) and which can be approximated in some sense by a

[1] In [HP93] and [EKJP98], completeness has to be understood in terms of the existence of a *unique* risk-neutral probability measure (in the spirit of the Second Fundamental Theorem of Asset Pricing). Additional borrowing constraints, however, make the notion of replicability (thus completeness) more complicated, and, for example, one has to distinguish between static and dynamic completeness.

sequence of products of uniformly integrable martingales and left-continuous decreasing processes. In the limit, we decompose the dual minimizer into a product of a decreasing process and an optional strong supermartingale deflator, where the decreasing part charges only at times when the associated optimal wealth process is zero. This result is closely related to a multiplicative version of the Doob-Meyer-Mertens decomposition for strong supermartingales; see [DM82, p. 414] for its additive version and [JS03, p. 138] for the multiplicative version, but only in the right-continuous case.

• In the paper "Optimal Consumption of Multiple Goods" [Mos18a], I consider the problem of optimal consumption of multiple goods. This problem was suggested to me in personal communication by Robert Merton during his visit to UConn - and it is now solved in incomplete semimartingale settings under weak model assumptions. I identified conditions that allow for the existence and uniqueness of the solution and provided a characterization of the optimal consumption strategy. I also illustrated the results with examples in both complete and incomplete models. In particular, I constructed closed-form solutions in particular incomplete models.

• In the paper "Necessary and Sufficient Conditions in the Problem of Optimal Investment with Intermediate Consumption" [Mos15], I consider several formulations of Merton's portfolio problem in the settings of a general incomplete semimartingale market, assuming that the utility is stochastic. As this problem is well-studied in the literature, my goal was to find *minimal* conditions on the model that ensure the validity of the key assertions of the utility maximization theory. I considered a framework that allowed studying in one formulation the problems of optimal investment from consumption over the finite and infinite time horizons, optimal investment from intermediate consumption and terminal wealth, optimal investment from terminal wealth, and optimal consumption from investment at discrete times.

The contributions include developing the theory in the general semimartingale settings and identifying the necessary and sufficient conditions needed for the key conclusions of the theory to be held. In order to prove the main theorems, I formulated their *abstract versions*, which allowed for the separation of the mathematical formulation from the financial issues. The construction of the dual problem, which is an auxiliary optimization problem that in many cases is easier to analyze than the primal one and which helps in the establishment of the properties of the primal problem, automatically gives the bipolar relations between the primal and dual domains and allows to prove conjugacy between the value functions.

• The subsequent work, "Optimal Investment with Intermediate Consumption and Random Endowment" [Mos17], is devoted to a similar framework that additionally includes random endowment. I establish the key assertions of the utility maximization theory, assuming *minimal* conditions on the financial market model and the random endowment. Besides, I propose alternative but equivalent conditions under which the conclusions of the theory hold.

Technically, the approach relies on an *increase of the dimensionality* of the value function, where the additional argument is the number of units of random endowment held in the portfolio. This shifts the proof ideas towards the multidimensional convex-analytic techniques combined with the classical convex duality method. For example, in order to show conjugacy between the value functions, I formulated auxiliary optimization problems, which permit the reduction of the dimensionality and (as a consequence) the reuse of the abstract theorems in my previous work [Mos15].

In addition to the usual conclusions of the utility maximization theory, it is possible to establish certain properties of the value functions on the boundaries of their domains, such as the upper semi-continuity of the primal value function and the lower semi-continuity of the dual value function. To achieve this, I use the direct method, see, e.g., [FL07], together with a technical proof of the uniform integrability of the positive part of the utility stochastic field composed with admissible consumption processes.

Indifference pricing

• In "Pricing of Contingent Claims in Large Markets" [MS24a], co-authored with Pietro Siorpaes (Imperial College London), we consider the problem of pricing in large markets in a framework where the large market limits the small ones with finitely many traded assets. We show that this framework allows accommodating indifference pricing in stochastic utility (also known as utility-based pricing) settings and arbitrage-free pricing. Adopting a stochastic integration theory with respect to a sequence of semimartingales, we introduce the notion of indifference prices for the large post-limit market and establish their existence, uniqueness, and relation to arbitrage-free prices. *In particular, we introduced the notion of indifference*

prices in the sense of [KK21, page 157] in large post-limit markets[2]. These results rely on a theorem of independent interest on utility maximization with a random endowment in a large market that we state and prove first. Further, we provide approximation results for utility-based and arbitrage-free prices in large markets by those in small markets. In particular, our framework allows for pricing the asymptotically replicable claims, where we also demonstrate the consistency of the pricing methodologies and provide positive examples[3].

• In "On Perturbations of Preferences and Indifference Price Invariance" [MT24], co-authored with Alexander Teplyaev (UConn), we investigated indifference pricing under perturbations of preferences in small and large markets. We establish stability results for small perturbations of preferences, where the latter can be stochastic. We obtain a sharp condition in terms of the associated concave and convex envelopes and provide counterexamples demonstrating that, without such an envelope condition, stability fails.

We also investigate a class of models where the indifference price does not depend on the preferences or the initial wealth. Here, under the existence of an equivalent separating measure, in the settings of deterministic preferences, we show that the class of indifference price invariant models is the class of models where the dual domain is stochastically dominant of the second order. We also provide a counterexample showing that, in general, this result does not hold over stochastic preferences; where instead, we show that indifference price invariant models are complete models (in both small and large markets). In the process, we establish a theorem of independent interest on the stability of the optimal investment problem under perturbations of preferences. Our results are new in both small and large markets, and thus, in particular, we introduce large stochastically dominant models, give examples of such models, and characterize them as indifference price invariant ones over deterministic preferences.

• A surprisingly nice result is obtained in an REU project "Representation of indifference prices on a finite probability space," [HFM24], which shows that, on a finite probability space, regardless of the choice of the utility function (deterministic or stochastic) and regardless of the particular form of the admissibility condition (which becomes technical in the infinite probability space settings, especially in large markets), the indifference price in the sense of [KK21, page 157] is unique (if the underlying utility is differentiable) and is given by the expectation of the discounted payoff under the dual-optimal measure.

• I am also currently trying to finish a single-authored paper, "An approach to the Greeks for indifference pricing," where I am polishing a 48-page draft, and I hope to upload it soon.

Stability of the Epstein-Zin problem

• In "Stability of the Epstein-Zin problem" [MM24], co-authored with Michael Monoyios (Oxford), we investigated the stability of the Epstein-Zin problem with respect to small distortions in the dynamics of the traded securities. We worked in incomplete market model settings, where our parametrization of perturbations allows for joint distortions in returns and volatility of the risky assets and the interest rate. Considering empirically the most relevant specifications of risk aversion and elasticity of intertemporal substitution, we provided a condition that guarantees the convexity of the domain of the underlying problem and results in the existence and uniqueness of a solution to it. Then, in the limit when the model perturbations vanish, relying on particular approximations of BSDEs with non-Liptschitz generators, we have proven the convergence of the optimal solutions and the associated value functions. Additionally, relying on an approximation and convergence of semimartingales in different topologies, we have proven convergence of the optimal wealth processes.

Stability of the indirect utility process

• In "Stability of the indirect utility process" [Mos21], I investigated the dynamic stability of the indirect utility process associated with a (possibly suboptimal) trading strategy under perturbations of the market. Establishing biconjugacy characterizations first, I have proven continuity and first-order convergence of the indirect-utility process under simultaneous perturbations of the finite variation and martingale parts of the stock price return.

On a technical level, I established versions of the Fenchel-Moreau and minimax theorems for non-compact subsets of a pair of non-locally convex topological vector spaces. The topic

[2]That is, the markets containing infinitely many traded assets.

[3]Another paper with Pietro Siorpaes, "Differentiation of Measures on an Arbitrary Measurable Space" [MS23] addresses some questions in analysis related to differentiation of measures on non-separable spaces with an application to convergence of martingales.

is closely related to characterizations of self-generating forward performance processes. I expect this paper to be instrumental in analyzing the forward indifference valuation, including the representations of the associated prices and the analysis of their stability under model perturbations.

Asymptotic analysis

• In the paper "Sensitivity analysis of the utility maximization problem with respect to model perturbations" [MS19], co-authored with Mihai Sîrbu (UT Austin), we studied the sensitivity of the expected utility maximization problem in a continuous semi-martingale market with respect to small changes in the market price of risk. Assuming that the preferences of a rational economic agent are modeled with a general utility function, we obtained a second-order expansion of the value function, a first-order approximation of the terminal wealth, and constructed trading strategies that match the indirect utility function up to the second order. If a risk-tolerance wealth process exists, using it as a numéraire and under an appropriate change of measure, we reduce the approximation problem to a Kunita-Watanabe decomposition.

• In "Asymptotic analysis of the expected utility maximization problem with respect to perturbations of the numéraire" [Mos20], in an incomplete model, where under an appropriate numéraire, the stock price process is driven by a sigma-bounded semimartingale, I investigated the sensitivity of the expected utility maximization problem to small perturbations of the numéraire. I established a second-order expansion of the value function and a first-order approximation of the terminal wealth. Relying on a description of the base return process in terms of its semimartingale characteristics, I also constructed wealth processes and corrections to optimal strategies that match the indirect utility function up to the second order. Finally, I related the asymptotic expansions to the existence of the risk-tolerance wealth process.

In this paper, I was able to leave the settings of a *continuous* stock-price (or controlled) process and to obtain results in general semimartingale settings. The corrections to optimal controls are given via representations of certain auxiliary martingales (under a change of measure and numéraire) in terms of predictable semimartingale characteristics. In particular, the results of this paper complement the ones in [KS06] by specifying corrections to trading strategies that match the indirect utility up to the second order.

• "An Expansion in the Model Space in the Context of Utility Maximization" [LMŽ18], co-authored with Kasper Larsen (Rutgers) and Gordan Žitković (UT Austin), analyzed the effect of the misspecifications in the parametric description of the stock evolution on the value function of a rational economic agent, whose preferences are described by a power utility with $p < 0$. This project is closely related to [MS19] and was finished earlier. In the framework of an incomplete financial market where the stock price is modeled by a continuous semimartingale, we performed an asymptotic analysis of the value function with respect to a small perturbation of the market price of risk. We established a second-order expansion formula and obtained corrections to optimal strategies that match the indirect utility up to the second order. Detailed examples, which are of separate interest, show the implications of our results, such as an approximation of the "less" tractable models by the "more" tractable ones. Besides, we establish a convergence result for a parametric family of continuous semimartingales with respect to a parameter. Besides, we establish a convergence result for a parametric family of continuous semimartingales with respect to a parameter.

• In "Quadratic expansions in optimal investment with respect to perturbations of the semimartingale model" [MS24b], co-authored with Mihai Sîrbu (UT Austin), we studied the response of the optimal investment problem to small changes in the stock price dynamics. Starting with a multidimensional semimartingale setting of an incomplete market, we suppose that the perturbation process is also a general semimartingale. We obtained second-order expansions of the value functions and first-order corrections to the optimizers and provided the adjustments to the optimal control that match the objective function up to the second order. We also illustrate the results with examples of base models that allow for closed-form solutions, but where this structure is lost under perturbations of the model where our results allow for an approximate solution.

In particular, apart from some contributions to mathematical finance, we have provided in the Appendix a characterization of the sets of orthogonal martingales (possibly with jumps) under simultaneous changes of measure and numéraire. I expect the results of this paper to be very helpful in a range of topics, from characterizations of various equilibria, particularly partial equilibria, to asymptotic analysis of indifference pricing.

Stochastic dominance for supermartingale deflators and related topics

• The paper "On the analyticity of the value function in optimal investment," [MSZ23] co-authored with Mihai Sîrbu (UT Austin) and Thaleia Zariphopoulou (UT Austin) contains the following probabilistic characterization of the set of supermartingale deflators (which is a generalization of the set of equivalent local martingale measures for the underlying stock price process): *the set of supermartingale deflators admits the maximal element in the sense of the infinite order stochastic dominance if and only if there is a maximal element in the sense of the second-order stochastic dominance.* This result holds under the assumption of no unbounded profit with bounded risk in the sense of [KK07] and possibly without an equivalent martingale measure.

Additionally, in this paper, we developed self-contained and mathematically rigorous characterizations of stochastic dominance of various orders, including the infinite one, in terms of the appropriate test functions. To the best of our knowledge, in the economics literature, such characterizations were obtained under mathematically restrictive conditions, at least for the topics of the next paragraph.

Consequences of the stochastic dominance for supermartingale deflators are further developed in [MSZ23]. Thus, with stochastic dominance, the higher-order differentiability of the value function and the optimizers can be established in multiple stochastic control problems. In particular, this allows us to go beyond the second-order asymptotics, which was a very difficult question. Please note that many papers on the asymptotic analysis in the context of the stochastic control problem, including the ones above and others, are technically very complicated, and they give expansions *up to the second order*. The question: "Can one say something about the higher-order derivatives of the optimizers and the value functions?" was hanging in the air of the mathematical finance community for years. In [MSZ23], we provided an affirmative answer to this question and established the analyticity of the value function in the expected utility maximization problem. Additionally, the analyticity relies on the complete monotonicity (that is, the existence of the derivatives of every order, such that these derivatives have alternating signs) of the inverse marginals of the utility function.

Counterexamples, which by themselves are of independent interest, show that the analyticity of the value function does not hold if either the utility function (even analytic) or the model does not belong to these special classes. More precisely, we show that for an arbitrary utility function, even completely monotonic, one can construct a market that is not stochastically dominant in the sense above, where the value function is not two times differentiable. The other counterexample shows that, even in a complete (therefore, stochastically dominant) market and with an analytic utility, asymptotic expansion results might fail without complete monotonicity of the utility function.

Investigations of Large Markets

As of now, I have three papers [Mos18b], [MS24a], and [MT24] which concern analysis of large markets. Above, I have briefly discussed [MS24a] and [MT24], which also deal with indifference pricing. Below, I will also discuss

• "Utility Maximization in the Large Markets" [Mos18b], where I analyzed a model of a large market with countably many traded assets and formulated a problem of the expected utility maximization. Assuming that the preferences of the economic agent are modeled with a stochastic utility and that the consumption occurs according to a stochastic clock, I establish the "usual" conclusions of the utility maximization theory, that is, the existence and uniqueness results, properties of the value functions and the optimizers. I also give a characterization of the value function in the large market in terms of a sequence of the value functions in the finite-dimensional models.

The notion of a trading strategy is based on the concept of a stochastic integral with respect to a sequence of semimartingales, introduced in [DDP06]. Mathematically, infinite-dimensional stochastic processes are more challenging to work with than their finite-dimensional counterparts. As a consequence of this, from the technical viewpoint, an approximation of the admissible consumptions in the large market by the ones in the finite-dimensional models requires a delicate truncation argument combined with localization. As semimartingales, in general, are not continuous, the main difficulty in this part of the work was constructing an approximating procedure that handles the jumps of the underlying processes.

I believe that the results in [Mos18b] and [MS24a] provide a convenient set of conditions for analyzing other problems in the settings of large markets. They include questions of the existence of partial equilibria and large deviation estimates. I plan to work on these topics.

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