# Assignment 5

1. Let elements of a matrix $K \in \mathbb{R}^{N\times N}$ be given by $K_{i,j} = \frac{ij}{i+j-1}$. Find the condition number of $K$ for n=8,10,12. You can use matlab.

2. Let $u_I$ be the piecewise linear interpolant of $u$ defined in class. Prove that

$$\|u' - u_I'\|_{L_2(0,1)} \leq Ch\|u''\|_{L_2(0,1)}$$

and

$$\|u - u_I\|_{L_2(0,1)} \leq Ch^2\|u''\|_{L_2(0,1)},$$

where $h$ is the mesh size.

3. Compute explicitly the local basis function on the interval $[x_j, x_{j+1}]$, for the piecewise quadratic Lagrange elements.

4. Using the piecewise linear finite element method, solve the following problem,

$$\begin{aligned} -u'' + u &= 1, \quad x \in (0,1), \\ u'(0) = u'(1) &= 0. \end{aligned}$$

Derive the exact solution and using it estimate the convergence rates at the node points.