Problem Set 6

1. Let K be a triangle with vertices $N_1 = (x_1, y_1)$, $N_2 = (x_2, y_2)$, and $N_3 = (x_3, y_3)$. Let λ_1, λ_2, λ_3 be the corresponding barycentric coordinates (basis functions).

   (a) Let $\mathbf{n_i}$ be the outward normal vector of the edge e_i and d_i be the distance from N_i to e_i. Show that
   $$\nabla \lambda_i = -\frac{\mathbf{n_i}}{d_i}, \quad i = 1, 2, 3.$$

   (b) Let θ_i be the angle associated with the vertex N_i. Show that
   $$\int_K \nabla \lambda_i \cdot \nabla \lambda_j dx = -\frac{\cot \theta_k}{2},$$
   where (i, j, k) is any permutation of $(1, 2, 3)$.

2. Write a matlab program that computes the solution to the following problem for arbitrary functions $f(x)$ $a(x) > 0$, $g_D(x)$ and $g_N(x)$ on a unit square $\Omega = [0, 1] \times [0, 1]$. You can use one point quadrature.
   $$\begin{aligned} -\nabla(a(x)\nabla u(x)) &= f(x), \quad x \in \Omega, \\ -\mathbf{n}(\mathbf{x}) \cdot (a(x)\nabla u(x)) &= k(u(x) - g_D(x)) - g_N(x) \quad x \in \partial\Omega, \end{aligned}$$
   where $k > 0$ is some fixed positive number

Due at the beginning of class on Thursday, October 26.