Problem Set 5

1. Give variational formulations and prove existence of solution of

$$-u''(x) = f(x) \quad x \in (0,1),$$

with the following boundary conditions

(a) $u(0) = u(1) = 0$,

(b) $u(0) = u'(1) = 0$,

(c) $-u'(0) + u(0) = u'(1) = 0$.

2. Determine the element mass and stiffness matrices for piecewise quadratic functions on an arbitrary triangle with corner coordinates (x_1, y_1), (x_2, y_2) and (x_3, y_3).

3. Let φ_1, φ_2 and φ_3 be the linear basis functions on element K. Prove the formula (3.64) from the textbook

$$\int_K \varphi_1^m(x)\varphi_2^n(x)\varphi_3^p(x)dx = \frac{2m!n!p!}{(m+n+p+2)!}|K|.$$

4. Show that the solution to

$$\begin{aligned} -\Delta u + u &= 0, \quad x \in \Omega, \\ n \cdot \nabla u &= g_N, \quad x \in \partial\Omega \end{aligned}$$

satisfies the stability estimate

$$\|u\|_{H^1(\Omega)} \leq C\|g_N\|_{L^2(\partial\Omega)}.$$

5. Write a matlab program that computes the solution to the following problem for an arbitrary function $f(x)$.

$$\begin{aligned} -u''(x) &= f(x), \quad x \in (0,1), \\ u(0) = u'(1) &= 0. \end{aligned}$$

Due at the beginning of class on Thursday, October 19.