MATH 5520 – Finite Element Methods October 5, 2017

Problem Set 4

1. Determine the basis functions for piecewise quadratic functions on an arbitrary triangle with corner coordinates (x_1, y_1), (x_2, y_2) and (x_3, y_3).

2. Compute the element mass and stiffness matrices on the triangle $\hat{K}$ with corners $(0, 0)$, $(1, 0)$ and $(0, 1)$.

3. Let V_h denote the Lagrange piecewise linear subspace on a quasi-uniform triangulation. Show the following Inverse Inequality

$$|\chi|_{W^{r,p}(D_h)} \leq Ch^{s-r+2/p-2/q}|\chi|_{W^{s,q}(D_h)} \text{ for } \chi \in V_h(D_h), \quad r > s, \quad 1 \leq p, q \leq \infty,$$

   where D_h is the closure of the union of any set of triangles in quasi-uniform triangulation of size h. Prove this inequality.

   Hint: First prove this inequality on a triangle K with vertices $(0, 0)$, $(h, 0)$, and $(0, h)$. This can be done by scaling to the reference triangle $\widehat{K}$ and using the fact that all norms on a fixed finite dimensional space are equivalent.

4. Let $f(x) = |x|^r$ for a given real number r. Prove that f has first order weak derivatives on the unit ball provided that $r > 1 - n$.

5. Suppose Ω be a unit disk in $\mathbb{R}^2$ and let p be a real number in the range $1 \leq p \leq \infty$. Prove that there is a constant C such that

$$\|v\|_{L^p(\partial\Omega)} \leq C\|v\|^{1-1/p}_{L^p(\Omega)}\|v\|^{1/p}_{W^{1,p}(\Omega)}$$

   for all $v \in W^{1,p}(\Omega)$. Explain what this means in the case $p = \infty$.

Due at the beginning of class on Thursday, October 12.