MATH 5520 – Finite Element Methods September 7, 2017

Problem Set 1

1. Let $\pi_h u$ be the piecewise linear interpolant of u defined in class. We proved that

$$\|u - \pi_h u\|_{L_2(x_i, x_{i+1})} \leq C h_i^2 \|u''\|_{L_2(x_i, x_{i+1})}.$$

   Obtain the explicit value of the constant C. Try to obtain one as small as possible.

2. (Problem 1.3 from the textbook.) Determine the linear interpolant $\pi_h f \in \mathbb{P}_1(I)$ on the single interval $I = [0, 1]$ to the following functions f.

   (a) $f(x) = x^2$.

   (b) $3\sin(2\pi x)$.

3. Find the exact solution of the following problem:

$$\begin{aligned} -u''(x) &= \frac{1}{x}, \quad x \in (0, 1), \\ u(0) = u(1) &= 0. \end{aligned}$$

   Is u a classical solution? Explain.

4. Find the local basis functions on the interval $[x_i, x_{i+1}]$, for the piecewise quadratic Lagrange elements.

5. Modify `L2Projector1D` to use 2 point Gauss quadrature instead and compute the L^2-projection $P_h f$ of the following functions f.

   (a) $f(x) = 1$.

   (b) $f(x) = x^3(x - 1)(1 - 2x)$.

   (c) $\arctan((x - 0.5)/\epsilon)$, with $\epsilon = 0.1$ and 0.01.

   Use a uniform mesh $\mathfrak{I}$ of the interval $I = [0, 1]$ with $n = 5$, 25 and 100 subintervals.

6. Write a matlab function `L2error1D` that computes the L^2 error between a continuous function f and a piecewise linear function $g \in V_h$ using 2 point Gauss quadrature. (The input should be the continuous function f and a n-vector g. The output should be the L^2-error.) Test your function with the function (b) from the previous problem.

Due at the beginning of class on Thursday, September 14.