Assignment 2 (T)

1. (Problem 2.1 from the textbook)

   Let Ω be a square with side 1. Show that

   $$\int_\Omega v^2 dx \leq \int_\Omega |\nabla v|^2 dx, \quad \forall v \in H_0^1(\Omega).$$

2. (Problem 2.5 from the textbook)

   Give a variational formulation of the inhomogeneous Neumann problem

   $$\begin{aligned} -\Delta u + u &= f \quad \text{in } \Omega \\ \frac{\partial u}{\partial n} &= g \quad \text{on } \partial\Omega \end{aligned}$$

   and check if the conditions (i)-(iv) of Section 2.1 are satisfied. Give a physical interpretation of such boundary condition.

3. (Problem 2.6 from the textbook)

   Give a variational formulation of the problem

   $$\begin{aligned} -\Delta u + u &= f \quad \text{in } \Omega \\ \gamma u + \frac{\partial u}{\partial n} &= g \quad \text{on } \partial\Omega, \end{aligned}$$

   where γ is a constant. When are the conditions (i)-(iv) of Section 2.1 satisfied? Give an interpretation of such boundary condition (sometimes called Robin boundary conditions).

4. (Problem 2.7 from the textbook)