Assignment 2 (C)

1. (Problem 2.1 from the textbook)

   Let Ω be a square with side 1. Show that

   $$\int_\Omega v^2 dx \le \int_\Omega |\nabla v|^2 dx, \quad \forall v \in H_0^1(\Omega).$$

2. (Problem 2.5 from the textbook)

   Give a variational formulation of the inhomogeneous Neumann problem

   $$\begin{aligned} -\Delta u + u &= f \quad \text{in } \Omega \\ \frac{\partial u}{\partial n} &= g \quad \text{on } \partial\Omega \end{aligned}$$

   and check if the conditions (i)-(iv) of Section 2.1 are satisfied. Give a physical interpretation of such boundary condition.

3. First, download the Matlab package AFEM@matlab (Google AFEM@matlab, follow the top link to the appropriate page of the Matlab file exchange, and download). Install the software by unzipping, putting the AFEM@matlab folder in an appropriate directory, and adding its path (including all subfolders) to the Matlab search path using the pathtool command within Matlab.

   Take a look at documentation by going to the 7 CodeMap directory in the AFEM@matlab folder, then opening index.html in a browser, or by opening AFEM@matlab.pdf in the home directory.

   We listed four main parts of a finite element code: the mesh generator, the matrix/vector assembly routine, the solver, and the postprocessing steps. Print out the files simple.m and Poisson.m (which is called by simple.m, and which contains most of the guts of the code), and identify where each of these parts of the code is located.

4. Modify your code from HW 1 to include the variable coefficients, i.e. your code should be able to solve the problem

   $$\begin{aligned} -(a(x)u'(x))' &= f(x), \quad 0 \le x \le 1 \\ u(0) &= u(1) = 0. \end{aligned}$$

   where $a(x)$ and $f(x)$ some functions. (You will need numerical quadrature to assemble the stiffness matrix A and the load vector F).