Assignment 1 (C)

1. (Problem 1.1 from the textbook)

   Show that if w is continuous on $[0,1]$ and

   $$\int_0^1 w(x)v(x)\,dx = 0, \quad \forall v \in V,$$

   then $w(x) = 0$ for $x \in [0,1]$. Here

   $$V = \{v :\ v \in C([0,1]),\ v' \text{ is piecewise continuous and bounded on } [0,1],\ \text{and } v(0) = v(1) = 0\}.$$

2. (Problem 1.3 from the textbook)

   Construct a finite-dimensional subspace V_h of V consisting of functions which are quadratic on each subinterval I_j of a partition of $I = (0,1)$. How can one choose the parameters to describe such functions? Find the corresponding basis functions. Then formulate a finite element method for the problem

   $$\begin{aligned} -u''(x) &= f(x), \quad 0 < x < 1 \\ u(0) &= u(1) = 0. \end{aligned} \tag{1}$$

   using the space V_h and write down the corresponding linear system of equations in case of a uniform partition.

3. (Computational problem)

   (a) Write a MATLAB code that solves the (1) with piecewise linear functions on the uniform grid with $h = 10^{-2}$ and $f \equiv 1$ and plot the solution.

   (b) Compute the exact solution to (1) and plot the error between the exact and the approximate solutions at the node points x_j, $j = 0, \ldots, 100$.

   (c) Subdivide each subinterval $I_j = [x_j, x_{j+1}]$ uniformly with 11 points i.e. $x_j = y_{j,0} < y_{j,1} < \cdots < y_{j,9} < y_{j,10} = x_{j+1}$, $j = 0, \ldots, 100$ and plot the error between the exact and the approximate solutions at all points $y_{j,i}$, $j = 0, \ldots, 100$, $i = 0, \ldots, 10$.

   (d) Try to explain what you have observed.