MATH 5510 – Numerical Analysis November 9, 2016

Assignment 7

1. Let $f : \mathbb{R} \to \mathbb{R}$ be $C^2(\mathbb{R})$. Assume $\bar{x}$ is a roof of $f(x)$ of multiplicity two, i.e. $f(\bar{x}) = f'(\bar{x}) = 0$ and $f''(\bar{x}) \neq 0$.

   (a) Show that the Newton's method $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ converges only linearly.

   (b) How can you modify the Newton's method to regain quadratic convergence?

2. Let $f : \mathbb{R} \to \mathbb{R}$ be $C^3(\mathbb{R})$. Design an iterative method for finding a root of $f(x)$ that has third order convergence.

   (Hint: Consider $g(x) = x - \frac{f(x)}{f'(x)} + h(x)\left(\frac{f(x)}{f'(x)}\right)^2$. Find a function $h(x)$ which may depend on $\frac{f(x)}{f'(x)}$ such that $g'(\bar{x}) = g''(\bar{x}) = 0$, where $\bar{x}$ is a root of $f(x)$.

3. One wants to solve the equation $x + \ln x$, whose root is near 0.5, using one or more of the methods:

$$(i)\ x_{k+1} = -\ln x_k \qquad (ii)\ x_{k+1} = e^{-x_k} \qquad (iii)\ x_{k+1} = \frac{x_k + e^{-x_k}}{2}$$

   (a) Which of the three methods can be used?

   (b) Which method should be used?

   (c) Give better iterative formula.

   Justify your answers.

4. Let $A \in \mathbb{R}^{n\times n}$ be a non-singular matrix. We are interested in computing A^{-1} by a fixed point iteration. Consider two fixed point iterations:

   (a) $X_{k+1} = X_k(I - AC) + C$, $k = 0, 1, 2, \ldots$, for some non-singular matrix $C \in \mathbb{R}^{n\times n}$,

   (b) $X_{k+1} = X_k(2I - AX_k)$, $k = 0, 1, 2, \ldots$

   What are the sufficient conditions for convegence in each case? Show that under these conditions $X_k \to A^{-1}$.