MATH 5510 – Numerical Analysis October 14, 2016

# Assignment 5

1. Let $p$-norm of the matrix $A : \mathbb{R}^m \to \mathbb{R}^n$ be defined by

$$\|A\|_p = \sup_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_p}, \quad \text{for} \quad 1 \leq p \leq \infty.$$

Show that

$$\|A\|_p = \sup_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_p} = \sup_{\|x\|_p = 1} \|Ax\|_p.$$

2. Show

$$\|A\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^{m} |a_{ij}|, \quad (\textit{maximum row norm}),$$

$$\|A\|_2 = \sqrt{\lambda_{max}(A^T A)}, \quad (\textit{spectral norm}),$$

where $\lambda_{max}(A^T A)$ is the largest eigenvalue of $A^T A$.

3. (a) Let

$$A = \begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Compute the matrix norms $\|A\|_p$ and the condition numbers $\kappa(A) = \|A\|_p \|A^{-1}\|_p$ for $p = 1, 2, \infty$.

(b) Let $b = (5, 1.02, 1.04, 1.1)^T$. Compute $\hat{b}$ by rounding the entries of $b$ to the nearest integers. Compute the solution $\hat{x}$ of $A\hat{x} = \hat{b}$.

(c) Use the result of the theorem proved in class to compute upper bounds for the relative errors

$$\frac{\|\hat{x} - x\|_p}{\|x\|_p}, \quad p = 1, 2, \infty.$$

Do not compute the solution $x$ of $Ax = b$.

(d) Compute the solution $x$ of $Ax = b$ and the relative errors

$$\frac{\|\hat{x} - x\|_p}{\|x\|_p}, \quad p = 1, 2, \infty.$$